The 26th Symposium of Complex Geometry
Kanazawa 2020
(Online)

	Nov.2	Nov.3	Nov.4	Nov.5
9:00-	zoom open	zoom open	zoom open	zoom open
9:30-10:30	Futaki	Ohsawa	Tsuji	Noguchi
10:40-11:40	Matsumura	R. Kobayashi	Inoue	Mabuchi
Lunch				
12:30-	zoom open	zoom open	zoom open	
13:00-14:00	Kasuya	N. Honda	Baba	
14:10-15:10	Koike	Yotsutani	Inayama	

今回は zoom を用いた online symposium となります.

Nov. 2

9:30- 10:30 Akito Futaki (Tsinghua University)
Title: Quantum moment map and obstructions to the existence of closed Fedosov star products

10:40-11:40 Shin-ichi Matsumura (Tohoku University)
Title: Hacon-McKernan's problem on rationally connectedness

13:00-14:00 Hisashi Kasuya (Osaka University)
Title: Non-invariant deformations of left-invariant complex structures on compact Lie groups

14:10-15:10 Takayuki Koike (Osaka City University)
Title: On the complement of a hypersurface with flat normal bundle which corresponds to a semipositive line bundle

Nov. 3

9:30- 10:30 Takeo Ohsawa (Nagoya University)
Title: A report on Hartogs type extension by the L^2-method from q-convex viewpoint

10:40-11:40 Ryoichi Kobayashi (Nagoya University)
Title: Nevanlinna Theory and Measure Concentration Phenomenon

13:00-14:00 Nobuhiro Honda (Tokyo Institute of Technology)
Title: セグレ曲面と種数１のミニツイスター空間
Segre surfaces and minitwistor spaces with genus one

14:10-15:10 Naoto Yotsutani (Kagawa University)
Title: Global smoothings of normal crossing surfaces with triple points

Nov. 4

9:30- 10:30 Hajime Tsuji (Sophia University)
Title: Structure of the moduli space of metrized canonical models

10:40-11:40 Eiji Inoue (University of Tokyo)
Title: μ-cscK metrics, μK-stability and a Lagrangian formalism

13:00-14:00 Shinpei Baba (Osaka University)
Title: Intersection of Poincare holonomy varieties and Bers’ simultaneous uniformization theorem

14:10-15:10 Takahiro Inayama (The University of Tokyo)
Title: From Hörmander's L^2-estimates to positivity

Nov. 5

9:30- 10:30 Junjiro Noguchi (The University of Tokyo)
Title: Bochner's Tube Theorem and Kashiwara's Local Version

10:40-11:40 Toshiki Mabuchi (Osaka University)
Title: Variational Methods in Complex Geometry

ABSTRACT

Nov. 2

Speaker: Akito Futaki (Tsinghua University)

Title: Quantum moment map and obstructions to the existence of closed Fedosov star products

abstract: It is shown that the normalized trace of Fedosov star product for quantum moment map depends only on the path component in the cohomology class of the symplectic form and the cohomology class of the closed formal 2-form required to define Fedosov connections. As an application we obtain a family of obstructions to the existence of closed Fedosov star products naturally attached to symplectic manifolds and Kähler manifolds. These obstructions are integral invariants depending only on the path component of the cohomology class of the symplectic form. Restricted to compact Kähler manifolds we re-discover an obstruction found earlier by La Fuente-Gravy. This last Kähler case can be treated similarly to the K-stability in the existence problem of cscK metrics. This talk is based on joint works with Laurent La Fuente-Gravy.

Speaker: Shin-ichi Matsumura (Tohoku University)

Title: Hacon-McKernan's problem on rationally connectedness

abstract: The famous result of Kollar-Miyaoka-Mori and Campana says that any smooth Fano varieties (=projective manifolds with ample anti-canonical bundle) are rationally connected (that is, any two points can be connected by a rational curve). Hacon-McKernan gave a problem that asks a relation between the geometry of rational curves and positivity of nef anti-canonical bundle for projective varieties with klt singularities, which was recently proved by Ejiri-Gongyo. In this talk, I will explain a sharper estimate than the problem posed by Hacon-McKernan and proved by Ejiri-Gongyo, by using positively curved singular hermitian metrics on direct image sheaves. A part of this talk is based on joint work with Junyan Cao and Frederic Campana.

Speaker: Hisashi Kasuya (Osaka University)

Title: Non-invariant deformations of left-invariant complex structures on compact Lie groups

abstract: It is known that every even dimensional compact Lie group admits a left(right)-invariant complex structure. The purpose of this talk is to give complex structures on even dimensional compact Lie groups which are not biholomorphic to any left(right)-invariant complex structure. We consider deformations of left-invariant complex structures on simply connected semisimple compact Lie groups which are a priori non-invariant. Computing their cohomologies, we show that they are not actually biholomorphic to left-invariant (right-invariant) complex structures. (joint work with Hiroaki Ishida (Kagoshima Univ.)

Speaker: Takayuki Koike (Osaka City University)
Title : On the complement of a hypersurface with flat normal bundle which corresponds to a semipositive line bundle

abstract: We investigate the complex analytic structure of the complement of a non-singular hypersurface with unitary flat normal bundle when the corresponding line bundle admits a Hermitian metric with semipositive curvature.

Nov. 3

Speaker: Takeo Ohsawa (Nagoya University)
Title: A report on Hartogs type extension by the L^2-method from q-convex viewpoint

abstract: Recently some progress has been made in the study of analytic families of $\mathbb{CP}^n \setminus \{pt\}$ by applying the L^2 method for the $\bar{\partial}$ operator (cf. [Oh-3,4]). As an application, the analyticity of $\mathbb{C}^n$-valued continuous functions with n-convex complements was obtained. After the submission of the manuscript of [Oh-4], the author was informed by N. Shcherbina that the latter analyticity result is contained in the thesis of T. Pawlaschyk [Pw] (see also [Pw-S]). This gave the author a motivation to continue [Oh-1] and [Oh-2] from the viewpoint of q-convexity. After writing a survey [Oh-5] on q-convex manifolds, he could generalize the main result of [Oh-2] in [Oh-6]. In the talk the results in [Oh-6] will be presented. Sketchy accounts of the proofs will be given, too.

For the convenience of the participants who are not familiar with topics in [Oh-1] and [Oh-2], a part of the introduction of [Oh-6] is presented below. Some of the notations and definitions will be explained in the talk.

A variant of Andreotti-Grauert's finiteness theorem on weakly 1-complete manifolds was obtained in [Oh-1] (see also [N-R] and [Ab]) and connected in [Oh-2] to an extension problem from submanifolds with semipositive normal bundles. This connection was suggested by Serre's celebrated works [S-1] and [S-2] on algebraic sheaves that translated the ideas of Oka-Cartan's theory in several complex variables into algebraic geometry. The method of L^2 estimates for the $\bar{\partial}$ operator was employed in [Oh-1] to explore such an analytic aspect of the sheaf cohomology. The point is that Andreotti-Grauert's finiteness theorem does not say anything directly about the effect of twisting the sheaves by line bundles, which was the main interest of [S-1,2], but Hörmander's theorem 3.4.9 in [H] does, although this advantage was not so explicitly stated there. Therefore, it seems worthwhile to pursue it further to look for a more general principle.

For that purpose, we shall modify a method of Hörmander in [H] in a way required in some geometric questions. More precisely, we shall refine it in such a way that we can study a class of L^2 $\bar{\partial}$-cohomology of certain 1-convex surfaces arising as compact complex surfaces minus smooth divisors whose self-intersection numbes are zero. Such a situation arises in the classification of the compactifications of $\mathbb{C}^* \times \mathbb{C}^*$ (cf. [U-1]), for instance. A general result by Ueda says that $S \setminus C$ is 1-convex if S is a compact complex surface with a smooth curve $C \subset S$ of finite type (cf. [U-2]). Accordingly, we would like to generalize the following Hartogs type extendability result to this case.

Theorem 0.1. (cf. [Oh-2, Theorem 1.4]) *Let M be a compact complex manifold of dimension n, let $E \to M$ be a holomorphic vector bundle and let D be an effective divisor on M. If the line bundle $[D]$ associated to D is semipositive and $E|_{|D|}$ is Nakano positive, then there exists a positive number μ_0 such that*

$$H^0(M, \mathscr{O}_M(K_M \otimes E \otimes [D]^\mu)) \to H^0(|D|, \mathscr{O}_D(K_M \otimes E \otimes [D]^\mu)$$

is surjective if $\mu \geq \mu_0$ and

$$H^{n,k}(M, E \otimes [D]^\mu) \cong H^{n,k}(M \setminus |D|, E))$$

if $k \geq 1$ and $\mu \geq \mu_0$. Here H^k and $H^{j,k}$ stand for the k-th sheaf cohomology and the $\bar{\partial}$-cohomology of type (j,k), respectively, K_M denotes the canonical line bundle of M,

For the validity of the consequence of Theorem 1.2 in [Oh-2], it suffices to assume that M is weakly 1-complete and E is Nakano positive outside a compact subset of $M \setminus |D|$ as long as $|D|$ is compact.

$|D|$ *the support of* D *and* $\mathscr{O}_X(\cdot)$ ($X = M$ or $|D|$) *the sheaf of the germs of holomorphic sections.*

A remarkable fact which strongly motivates a generalization of Theorem 0.1 is that, for any embedding $C \hookrightarrow S$ of finite type, the bundle $[C]$ is never semipositive on S whenever $S \setminus C$ is 1-convex. (See [Ko-2] for a detailed analysis of the semipositive case.)

A general result which helps to say something also for such cases turned out to be the following.

THEOREM 0.2. *Let* (X, g) *be a complete Hermitian manifold of dimension* n *and let* (E, h) *be a holomorphic Hermitian vector bundle over* X *such that* (X, g, E, h) *is* q*-elliptic at infinity. Assume that* X *is equipped with a positive* C^∞ *exhaustion function* Φ *satisfying*

$$\sup\{\Phi(x); (\partial\bar{\partial}\Phi)_q(x) < 0\} < \infty$$

and

$$\lim_{c\to\infty} \inf\{(\partial\bar{\partial}\Phi - \Phi^{-1}\partial\Phi\bar{\partial}\Phi)_q(x); \Phi(x) > c\} \geq 0.$$

Then the following a) and b) hold.

a) The E*-valued* L^2 $\bar{\partial}$*-cohomology group* $H^{n,k}_{(2),\Phi}(X, E)$ *of* X *with respect to* $(g, he^{-\Phi})$ *is mapped for all* $k \geq q$ *bijectively onto* $H^{n,k}(X, E)$ *by the homomorphism induced from the inclusion. Moreover, the map* $H^{n,q-1}_{(2),\Phi}(X, E) \to H^{n,q-1}(X, E)$ *has a dense image.*

b) If moreover

$$\lim_{c\to\infty} \inf\{\Phi(x)^{1+\epsilon}(\partial\bar{\partial}\log\Phi)_q(x); \Phi(x) > c\} \geq 0 \tag{1}$$

holds for some $\epsilon > 0$, *then the* L^2 *cohomology groups* $H^{n,k}_{(2),\mu\log\Phi}(X, E)$ *are isomorphic to* $H^{n,k}(X, E)$ *for* $k \geq q$ *if* μ *is sufficiently large, and the map*

$$\varinjlim_{\mu} H^{n,q-1}_{(2),\mu\log\Phi}(X, E) \to H^{n,q-1}(X, E)$$

has a dense image. Here μ *runs through* $\mathbb{N}$.

That $\varinjlim_{\mu} H^{n,0}(M, E \otimes [D]^\mu) \to H^{n,0}(M \setminus |D|, E)$ is dense in $H^{n,0}(M \setminus |D|, E)$ is also contained in the proof of Theorem 0.1.

The proof of Theorem 0.2 is based on a method of approximation which is a variant of Hörmander's method in the spirit of [Oh-T].

By specializing Theorem 0.2, Theorem 0.1 is generalized as follows.

THEOREM 0.3. *Let M be a weakly 1-complete manifold of dimension n, let $E \to M$ be a holomorphic vector bundle and let D be an effective divisor with compact support. Assume that $[D]|_{|D|}$ is semipositive and $E|_{M\setminus K}$ is Nakano positive for some compact subset K of $M \setminus |D|$. Then multiplication by a canonical section of $[D]$ induce isomorphisms between $H^{n,k}(M, E\otimes[D]^{\mu-1})$ and $H^{n,k}(M, E\otimes[D]^{\mu})$ $(k \geq 1)$ for sufficiently large μ. In particular*

$$H^k(M, \mathscr{O}_M(K_M \otimes E \otimes [D]^\mu)) \to H^k(|D|, \mathscr{O}_D(K_M \otimes E \otimes [D]^\mu))$$

is surjective for sufficiently large μ and for all k. Moreover, if D is a pseudoconcave divisor of order >1 , then

$$H^{n,k}(M, E \otimes [D]^\mu) \cong H^{n,k}(M \setminus |D|, E), \quad k \geq 1$$

hold for sufficiently large μ and the set of meromorphic sections of $K_M \otimes E$ with poles along $|D|$ is dense in $H^0(M \setminus |D|, \mathscr{O}_M(K_M \otimes E))$.

COROLLARY 0.4. *Let M be a connected compact complex manifold. If there exist an effective divisor $D \neq 0$ on M and a holomorphic line bundle $B \to M$ such that $[D]|_{|D|}$ is semipositive and $B|_{|D|}$ is positive. Then M is a Moishezon manifold.*

Corollary 0.1 was proved in [Oh-2] under a stronger assumption that $[D]$ is semipositive on M.

COROLLARY 0.5. *Let M be a connected weakly 1-complete Kähler manifold, let (E, h) be a Nakano semipositive vector bundle over M and let $D(\neq 0)$ be a pseudoconcave divisor on M of order >1 such that $|D|$ is compact and the curvature form of h is Nakano positive on $M \setminus K$ for some compact set $K \subset M \setminus |D|$. Then $H^{n,k}(M, E \otimes [D]^\mu) = 0$ $(k \geq 1)$ for sufficiently large μ.*

For the question arising from Ueda's theory, the following is an answer.

THEOREM 0.6. *Let S be a compact complex surface and let $C \subset S$ be a complex curve such that* $\deg([C]|C) \geq 0$. *Then, for any holomorphic vector bundle $E \to S$ such that $E|_C$ is positive, there exists a positive number μ_0 such that*

$$H^k(S, \mathscr{O}_S(K_S \otimes E \otimes [C]^{\mu-1})) \cong H^k(S, \mathscr{O}_S(K_S \otimes E \otimes [C]^{\mu}))$$

canonically if $\mu \geq \mu_0$. In particular, the maps

$$H^0(S, \mathscr{O}_S(K_S \otimes E \otimes [C]^{\mu})) \to H^0(C, \mathscr{O}_C(K_S \otimes E \otimes [C]^{\mu})) \ \ (k = 0, 1)$$

are surjective if $\mu \geq \mu_0$. If moreover the embeddinng $C \hookrightarrow S$ is of finite type, then

$$H^{2,k}(S, E \otimes [C]^{\mu}) \cong H^{2,k}(S \setminus C, E)) \ \ (k = 1, 2)$$

holds for sufficiently large μ and the set of meromorphic sections of $K_S \otimes E$ with poles along C is dense in $H^0(S \setminus C, \mathscr{O}(K_S \otimes E))$.

COROLLARY 0.7. *Let S be a connected compact complex surface, let $C \subset S$ be a smooth complex curve of finite type and let $B \to S$ be a holomorphic line bundle such that $B|_C$ is positive. Then B is big.*

Dano Kim has shown an example of nef, big and non semipositive line bundle on a nonsingular projective surface (cf. [Fn, Example 5.2]). See also [Ko-1] .

COROLLARY 0.8. *A connected compact complex surface is projective algebraic if and only if it contains a smooth curve of genus ≥ 2 with semipositive normal bundle.*

COROLLARY 0.9. *Let S be a connected complex surface with a complex curve $C \subset S$ of finite type. Then, for any holomorphic line bundle $B \to S$ with* $\deg(B|_C) > 0$, *one can find $\mu \in \mathbb{N}$ and $s_0, s_1, s_2, s_3, s_4, s_5 \in H^0(S, \mathscr{O}(K_S \otimes B \otimes [C]^{\mu}))$ such that $\bigcap_{k=0}^{5} s_k^{-1}(0) \subset S \setminus C$ and $(s_0 : s_1 : s_2 : s_3 : s_4 : s_5)$ embeds $S \setminus C \setminus \bigcap_{k=0}^{5} s_k^{-1}(0)$ into $\mathbb{CP}^5$.*

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Speaker: Ryoichi Kobayashi (Nagoya University)

Title: Nevanlinna Theory and Measure Concentration Phenomenon

abstract: 射影代数多様体上の豊富直線束のテンソル冪を上げていくと，際限なく次元の大きい射影空間への小平埋め込みの列が現れる．こういう列に対する漸近解析は，幾何学的量子化によく現われる．一方，高い次元の射影空間や付随するグラスマン多様体では，測度集中現象が発生する．幾何学的量子化の設定は漸近的であり，必然的に測度集中現象が伴っている．これがどういう意味を持つかを問う問題は興味深い．本講演では，巨大次元の射映空間への小平埋め込みに伴う測度集中現象が，一般の射影代数多様体への正則曲線の値分布の研究にどんな効果をもたらすかについて述べる．これは林培強君との共同研究に基づく講演である．

Speaker: Nobuhiro Honda (Tokyo Institute of Technology)

Title: セグレ曲面と種数 1 のミニツイスター空間

Segre surfaces and minitwistor spaces with genus one

abstract: 自己交点数が 2 の非特異有理曲線をもつ複素曲面はミニツイスター空間とよばれる。これらの有理曲線のパラメーター空間は 3 次元複素多様体になり、Einstein-Weyl 構造とよばれる微分幾何的な構造をもつことが知られている。10 年ほど前に中田文憲さん（現福島大）との共同研究で、上記有理曲線たちに通常二重点を持たせても同様の結果が成立することを示した。通常二重点の個数のことをミニツイスター空間の種数という。本講演では、種数 1 のミニツイスター空間がセグレ曲面と呼ばれる 4 次元射影空間の中の

4 次曲面たちに他ならないこと、対応する Einstein-Wey l 空間がその射影的双対多様体の Zariski 開集合になることを示す。詳細は arXiv:2009.05242 にある。

Speaker: Naoto Yotsutani (Kagawa University)

Title: Global smoothings of normal crossing surfaces with triple points

abstract: Let X be a normal crossing compact complex surface with triple points. In a differential geometrical way, we prove that there exists a family of smoothings of X when X satisfies suitable conditions. Furthermore, we construct an explicit example of a normal crossing variety with triple loci which admits a d-semistable degeneration. This is a joint work with M. Doi based on his unpublished work.

Nov. 4

Speaker: Hajime Tsuji (Sophia University)

Title: Structure of the moduli space of metrized canonical models

abstract: In this talk I would like to present how to endow the complex structure and algebraic structure on the moduli space of metrized canonical models and study the hyperbolicity of it.

Speaker: Eiji Inoue (The University of Tokyo)

Title: μ-cscK metrics, μK-stability and a Lagrangian formalism

abstract: I originally started up a study on μ-cscK metrics and μK-stability of polarized manifolds motivated by the moment map picture on Kahler-Ricci solitons. In this talk, after reviewing some backgrounds, I will introduce rather different new perspectives on μ-cscK metrics and μK-stability. The starting point is a Lagrangian formalism on Perelman's W-entropy. It will lead us to a conjectural picture on optimal destabilization, which is reminiscent of He, Dervan-Szekelyhidi, Hisamoto and Han-Li's work on Chen-Sun-Wang's degeneration along Kahler-Ricci flow.

Speaker: Shinpei Baba (Osaka University)

Title: Intersection of Poincare holonomy varieties and Bers’ simultaneous uniformization theorem.

abstract: Given a compact Riemann surface X, the space of holomorphic quadratic differentials on X is a complex vector space. By the holonomy map for complex projective structures on X, this vector space properly embeds onto a half-dimensional analytic submanifold of the PSL(2, $\mathbb{C}$)-character variety, the space of representations of the fundamental group of X into PSL(2, $\mathbb{C}$).

Given two diffeomorphic Riemann surfaces structures, we have two such sub-manifolds in the character variety, and in this talk, we discuss their intersection. In order to understand the intersection, we utilize a cut-and-paste operation, called grafting, of projective structures on Riemann surfaces.

Speaker: Takahiro Inayama (The University of Tokyo)

Title: From Hörmander's L^2-estimates to positivity

abstract: In this talk, using a twisted version of Hörmander's L^2-estimates, we give new characterizations of notions of positivity and partial positivity. By using these characterizations, we discuss definitions of Nakano semi- positivity and uniform q-semi-positivity for singular Hermitian metrics and show some applications. We also prove vanishing theorems, which generalize both Nakano type and Demailly-Nadel type vanishing theorems.

Nov. 5

Speaker: Junjiro Noguchi (The University of Tokyo)

Title: Bochner's Tube Theorem and Kashiwara's Local Version

abstract:

序. 多変数関数論における領域の凸性について

$$(\text{アファイン})\text{凸} \subset \text{多項式凸} \subset \text{正則凸}$$

という包含関係がある．これらが一致する領域のクラスとして**管領域** (tube domain) と呼ばれる領域がある．今回は，これを廻る話しで，'解析接続' がキーワードである．

§1. 領域 $R \subset \mathbb{R}^n$ を実部とし虚部は $\mathbb{R}^n$ 全体として "$T_R := R + i\mathbb{R}^n(\subset \mathbb{C}^n)$" と定義される領域を管領域という．$R$ を T_R の (実) 基底という．多変数 $w_1, \ldots, w_n$ の巾級数 $f(w) = \sum_{\alpha \in \mathbb{Z}_+^n} c_\alpha w^\alpha$ の収束域を調べる為に $w_j = e^{z_j}$，$z_j = x_j + iy_j$ とおくと y_j はなんでもよいので (z_j) の領域としては管領域 $T_R = R + i\mathbb{R}^n$ になり，f は，R の凸包 $\mathrm{co}(R)$ を実基底とする管領域 $T_{\mathrm{co}(R)}$ で収束し，それが収束の最大領域であることが古く知られている

(Hartogs). これを，(y_j) について周期的でない一般の関数に拡張したのが次の **Bochner の定理** (Stein $n = 2$) である．

THEOREM 0.10 (Bochner 1937/38, K. Stein ($n = 2$) 1937)). 管領域 $\Omega = R + i\mathbb{R}^n$ 上の任意の正則関数は，R の (アファイン) 凸包 $\mathrm{co}(R)$ を実基底とする管領域 $\mathrm{co}(\Omega) = \mathrm{co}(A) + i\mathbb{R}^n$ まで解析接続される (正則凸包).

目標 1 定理 0.10 の簡短証明を与える．

正則包 $\hat{\Omega}$ (一般には $\mathbb{C}^n$ 上の複葉領域になる) を考え，岡の境界距離定理 ($-\log\delta(p, \partial\hat{\Omega})$ が多重劣調和) を使う．

§2. 虚部の $\mathbb{R}^n$ を有限に切って開球 B や多重円板 $\mathrm{P}\Delta$ にして有限管領域 $\Omega = R + iB$ や $\Omega = R + i\mathrm{P}\Delta$ を考える．

目標 2 有限管領域の正則包は単葉とは限らない．例の構成.

PROPOSITION 0.11. ある領域 $A \subset \mathbb{R}^2$ と開球 $B \subset \mathbb{R}^2$ をとると，$\nu(\leq \infty,$ 任意$)$ 葉の不分岐被拡領域 $\pi_\nu : \Omega_\nu \to \mathbb{R}^2 + iB$ が存在して，Ω_ν は正則領域である．

真に複葉な無限管被拡領域は，正則分離，特に正則領域ではない (阿部誠の定理 1985) ので，無限管と有限管ではだいぶ事情が異なることになる．

§3. 上述の有限管領域を原点の近くで考えるのが**柏原の局所化版**である．実解析的多様体 M の複素化 X をとる．M の点を境界に持つ開集合上の正則関数の M 上の境界値を考えるのに M に沿った実モノイダル変換て考える (層 $\mathscr{C}$ の理論). それで十分であることが正則関数が虚部についてはある錐上に解析接続されるという次の補題で担保される (佐藤の超関数論，SKK 1970). 以下つごう上，実部と虚部を入れ替える．

LEMMA 0.12 (柏原 1970, 小松 (彦) 1972). $x = (x_1, x') \in \mathbb{R}^n$ $(x_1 \in \mathbb{R}, x' \in \mathbb{R}^{n-1})$, $y \in \mathbb{R}^n$ として

$$A_1 := \{x = (x_1, x') \in \mathbb{R}^n : 0 < x_1 < a, x' = 0\}, \qquad B = \{\|y\| < R\}$$

による集合 $A_1 + iB$ を含む開集合 Ω で正則な関数は，原点を頂点とする局所的な開錐

$$\{(z_j) \in \mathbb{C}^n : 0 < x_1 < \varepsilon,\ \|x'\| < \varepsilon x_1,\ \|y\| < \varepsilon\}$$

上に解析接続される．ここに，$\varepsilon > 0$ は U のみによる．

目標 3 この $\varepsilon > 0$ を決めたい.

A_1 を含む開集合 $V \subset \{(x_1, x') : 0 < x_1 < a, x' \in \mathbb{R}^{n-1}\}$ をとり $\Omega = V + iB$ を考える. $0 < \tau < 1$ に対し V_τ を $V \cap \{(x_1, x') : 0 < x_1 < r, \|x'\| < (1-\tau)R\}$ の A_1 を含む連結成分とする.

$$\tilde{\Omega} = \bigcup_{0<\tau<1} \big(\mathrm{co}(V_\tau) + i\tau B\big). \tag{2}$$

THEOREM 0.13. Ω の正則包は, $\tilde{\Omega}$ を単葉領域として含む.

注意. 各 τ について $\mathrm{co}(V_\tau) + i\tau B$ は $0 + i\tau B$ の点を頂点とする開錐である.

Reference:

J. Noguchi, A brief proof of Bochner's tube theorem and Kashiwara's local version, arXiev 2020.

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Title: Variational Methods in Complex Geometry

abstract: The K-energy and the Ding energy play very important roles in the existence problem of canonical Kähler metrics. For another problem in complex geometry, by cooking up a suitable functional, we study the problem from a purely variational point of view.