ON THE COHOMOLOGY VANISHING WITH POLYNOMIAL GROWTH ON COMPLEX MANIFOLDS WITH PSEUDOCONVEX BOUNDARY

TAKEO OHSAWA

Dedicated to Tetsuo Ueda on his seventieth birthday

Abstract. $\bar\partial$ cohomology groups with polynomial growth $H^{r,s}_{p.g.}$ will be studied. It will be shown that, given a complex manifold M, a locally pseudoconvex bounded domain $\Omega \Subset M$ satisfying certain geometric boundary condition and a holomorphic vector bundle $E \to M$, $H^{r,s}_{p.g.}(\Omega, E) = 0$ holds for all $s \geq 1$ if E is Nakano positive and $r = \dim M$. It will be also shown that $H^{r,s}_{p.g.}(\Omega, E) = 0$ for all r and s with $r + s > \dim M$ if moreover rank$E = 1$. By Deligne-Maltsiniotis-Sasakura's comparison theorem, it follows in particular that, for any smooth projective variety X, for any ample line bundle $L \to X$ and for any effective divisor D on X such that $[D]|_{|D|} \geq 0$, the algebraic cohomology $H^s_{alg}(X \setminus |D|, \Omega^r_X(L))$ vanishes if $r + s > \dim X$.

Introduction

This is a continuation of the series of papers [Oh-2∼5]. The motivation of [Oh-2] was to apply a method of Hörmander in [H] to an extension problem on a compact complex manifold M with a holomorphic vector bundle $E \to M$ and an effective divisor D. It was proved in [Oh-2] that the natural restriction map $H^0(M, \mathcal{O}(K_M \otimes E \otimes [D]^\mu)) \to H^0(|D|, \mathcal{O}_D(K_M \otimes E \otimes [D]^\mu))$ is surjective for sufficiently large μ if $E|_{|D|}$ is positive and $[D]$ is semipositive, where K_M denotes the canonical bundle of M. The proof is based on the isomorphisms between the L^2 cohomology groups $H^{n,s}_{(2)}(M, E, g, he^{-\mu\varphi})$ and $H^{n,s}_{(2)}(M, E, g, he^{-(\mu+1)\varphi})$ $(s \geq 1)$, where $n = \dim M$, g is a complete metric on M, h is a fiber metric of E and φ is a plurisubharmonic exhaustion function on $M \setminus |D|$

2010 *Mathematics Subject Classification.* Primary 32E40; Secondary 32T05 .

Key words and phrases. cohomology with polynomial growth, UBS domain, L^2 $\bar\partial$-cohomology, vanishing theorem.

The author thanks to Shigeru Takeuchi, Shin-ichi Matsumura and Tadasi Ashikaga for making useful comments. In particular, Matsumura brought the paper [L-R-W] under the author's attention.

which is of logarithmic growth near $|D|$. The isomorphism is a variant of Proposition 3.4.5 in [H] (cf. [Oh-1] and [Nk-R]). A somewhat intriguing aspect of this extension theorem is that n is arbitrary although the situation is quite analogous to the Bochner-Hartogs type extension rather than that of Oka and Cartan. In this situation, it seems worthwhile to generalize the result by replacing the assumption $[D] \geq 0$ by $[D]|_{|D|} \geq 0$. We note that there are interesting cases where $[D] \ngeq 0$ but $[D]|_{|D|} \geq 0$ and that the neighborhoods of such D have been studied in detail when $\dim M = 2$ and D is smooth by Ueda [U] and Koike [K]. In [Oh-3], it turned out that one can replace the condition $[D] \geq 0$ by $[D]|_{|D|} \geq 0$ by modifying Hörmander's technique in such a way that some non-plurisubharmonic function works as φ to establish $H^{n,s}_{(2)}(M, E, g, he^{-\mu\varphi}) \cong H^{n,s}_{(2)}(M, E, g, he^{-(\mu+1)\varphi})$ $(s \geq 1)$ for $\mu \gg 1$. By this refinement of Hörmander's method, a bundle-convexity theorem has been obtained in [Oh-4] for a class of locally pseudoconvex domains. An approximation theorem obtained in [O-3] was extended in [Oh-5] to a more restricted class of domains than in [Oh-4]. The purpose of the present article is to show in the same vein that the celebrated vanishing theorems of Akizuki-Kodaira-Nakano naturally extend to this latter class.

Namely, we consider a locally pseudoconvex bounded domain Ω in a (not necessarily compact but connected) complex manifold M and a holomorphic vector bundle $E \to M$. In what follows we assume that every connected component of $\partial\Omega$ is either a C^2-smooth real hypersurface or the support of an effective divisor. For simplicity we assume that each hypersurface component of $\partial\Omega$ separates M into two connected components, since one may take a double covering each time a nonseparating component of $\partial\Omega$ appears. In [Oh-4] we considered those Ω such that the divisorial components of $\partial\Omega$ are supported on an effective divisor whose normal bundles are semipositive. Such Ω will be called ***weakly pseudoconvex domains of regular type*** and we set $\partial_{hyp}\Omega := \partial\overline{\Omega}$ and $\partial_{div}\Omega := \partial\Omega \setminus \partial_{hyp}\Omega$. We shall restrict ourselves below to a smaller class of domains by imposing a condition on $\partial_{hyp}\Omega$.

A weakly pseudoconvex domain of regular type Ω will be called a ***weakly pseudoconvex domain of very regular type*** if $\partial_{hyp}\Omega$ is of class C^3 and the Levi form of the defining function say ρ of $\overline{\Omega}^{\circ}$ satisfies

$$\partial\bar{\partial}\rho \geq O(\rho^2)$$

on a neighborhood of $\partial_{hyp}\Omega$. It will also be called a ***UBS-domain*** in short, since the conditions on $\partial\Omega$ come from the works of Ueda [U] and Boas-Straube [B-S].

We shall prove the following.

Theorem 0.1. *In the above situation, assume that Ω is a UBS domain and E is Nakano positive. Then*
$H^{r,s}_{p.g.}(\Omega, E) = 0$ *for all* $s \geq 1$ *if* $r = \dim M$
(see §1 for the definition of $H^{r,s}_{p.g.}(\Omega, E)$*) and*
$H^{r,s}_{p.g.}(\Omega, E) = 0$ *for all* r *and* s *with* $r + s > \dim M$ *if* $\mathrm{rank} E = 1$.

By Deligne-Maltsiniotis-Sasakura's comparison theorem asserting the equivalence of the cohomology of polynomial growth and algebraic cohomology on quasi-projective varieties, one has the following in particular.

Corollary 0.1. *For any* n*-dimensional smooth projective variety* X*, for any Nakano positive vector bundle* $E \to X$ *and for any effective divisor* D *on* X *such that* $[D]|_{|D|} \geq 0$*, the algebraic cohomology group* $H^s_{alg}(X \setminus |D|, \Omega^n_X(E))$ *vanishes for* $s \geq 1$*, where* Ω^r_X *denotes the sheaf of holomorphic* r *forms. If moreover* $\mathrm{rank} E = 1$, $H^s_{alg}(X \setminus |D|, \Omega^r_X(E)) = 0$ *for* $r + s > n$.

Here we mean by $[D]|_{|D|} \geq 0$ that the line bundle $[D]$ admits a fiber metric whose curvature form is semipositive when it is restricted to the Zariski tangent spaces of $|D|$ (and no furhter semipositivity is assumed on the finite neighborhoods of $|D|$).

We note that an analogous vanishing for $H^s(X, \Omega^r_X(\log D) \otimes L)$ was proved by Norimatsu [N] by combining Akizuki-Nakano's vanishing theorem and Deligne's filtration of $\Omega^r_X(\log D)$ in [D-1] by assuming that D is a divisor of simple normal crossings. Recently, a vanishing theorem for $H^s(X, \Omega^r_X(\log D))$ was obtained by Liu, Wan and Yang in [L-W-Y] by combining a vanishing for H_{alg} with [D-2] while the Norimatsu vanishing for $H^s(X, \Omega^r_X(\log D) \otimes L)$ was extended by Liu, Rao and Wan in [L-R-W] by the standard L^2 method.

1. Cohomology with polynomial growth

After recalling the basic notations, $\bar\partial$ cohomology groups with polynomial growth will be defined.

Let M be a connected complex manifold equipped with a Hermitian metric g and let $E \to M$ be a holomorphic Hermitian vector bundle with a C^∞ fiber metric h. For any continuous function $\varphi : M \to \mathbb{R}$, we denote by $L^{r,s}_{(2)}(M, E, g, he^{-\varphi})$ the set of square integrable E-valued (r, s)-forms with respect to $(g, he^{-\varphi})$. For simplicity we shall often abbreviate $L^{r,s}_{(2)}(M, E, g, he^{-\varphi})$ as $L^{r,s}_{(2),\varphi}(M, E)$. By $C^{r,s}(M, E)$ we denote the set of C^∞ (r, s)-forms on M with values in E and by

$$\bar\partial : C^{r,s}(M, E) \to C^{r,s+1}(M, E)$$

the complex exterior derivative of type $(0,1)$. We put

$$C_0^{r,s}(M,E) = \{u \in C^{r,s}(M,E); \mathrm{supp} u \Subset M\}$$

and denote also by $\bar\partial$ the maximal closed extension of $\bar\partial|_{C_0^{r,s}(M,E)}$ as a linear operator from $L^{r,s}_{(2),\varphi}(M,E)$ to $L^{r,s+1}_{(2),\varphi}(M,E)$. Namely, the domain of the operator $\bar\partial : L^{r,s}_{(2),\varphi}(M,E) \to L^{r,s+1}_{(2),\varphi}(M,E)$ is defined as

$$\{u \in L^{r,s}_{(2),\varphi}(M,E); \bar\partial u \in L^{r,s+1}_{(2),\varphi}(M,E)\},$$

where $\bar\partial u$ is defined in the sense of distribution.

Then we put

$$H^{r,s}_{(2)}(M,E,g,he^{-\varphi}) = \frac{\mathrm{Ker}\bar\partial \cap L^{r,s}_{(2)}(M,E,g,he^{-\varphi})}{\bar\partial(L^{r,s-1}_{(2)}(M,E,g,he^{-\varphi})) \cap L^{r,s}_{(2)}(M,E,g,he^{-\varphi})}.$$

Given a bounded domain $\Omega \Subset M$ we put

$$\delta(z) = \delta_\Omega(z) := \mathrm{dist}_g(z, M \setminus \Omega) \quad (z \in \Omega).$$

Here $\mathrm{dist}_g(A,B)$ denotes the distance between A and B with respect to g. Then $H^{r,s}_{p.g.}(\Omega,E)$, the E-valued $\bar\partial$ cohomology group of Ω of type (r,s) with polynomial growth, is defined as the inductive limit of $H^{r,s}_{(2)}(\Omega,E,g,h\delta^\mu)$ as $\mu \to \infty$. Clearly $H^{r,s}_{(2)}(\Omega,E,g,h\delta^\mu)$ and $H^{r,s}_{p.g.}(\Omega,E)$ do not depend on the choices of g and h.

The most basic fact on $H^{r,s}_{p.g.}(\Omega,E)$ is the following, which is a direct consequence of the combination of [H, Theorem 2.2.3] with Oka's lemma asserting the plurisubharmonicity of $\log\frac{1}{\delta_\Omega}$ for any pseudoconvex domain $\Omega \subset \mathbb{C}^n$, with respect to the Euclidean metric. Although Theorem 2.2.3 is only stated when E is the trivial bundle, the proof of the general case is similar.

Theorem 1.1. *For any bounded pseudoconvex domain Ω in $\mathbb{C}^n$ and for any holomorphic Hermitian vector bundle E on a neighborhood of $\overline{\Omega}$,*

$$H^{r,s}_{p.g.}(\Omega,E) = 0 \quad \text{for all } r \geq 0 \text{ and } s \geq 1.$$

Based on Theorem 1.1, combining Cauchy's estimate with the canonical equivalence between Dolbeault and Čech cohomology, one has the following.

Theorem 1.2. *For any smooth projective algebraic variety X, for any algebraic vector bundle $E \to X$ and for any analytic set $D \subset X$ of codimension one, $H^{r,s}_{p.g}(X \setminus D, E)$ is canonically isomorphic to the corresponding algebraic cohomology group $H^s_{alg}(X \setminus D, \Omega^r_X(E))$ for any r and s.*

We note that Theorem 1.2 is naturally extended to the equivalence between the cohomology groups $H_{p.g.}$ and H_{alg} with coefficients in coherent algebraic sheaves over quasi-projective algebraic varieties (cf. [S]).

2. Vanishing of $H^{r,s}_{(2)}$

Let us recall a general vanishing theorem for those $H^{r,s}_{(2)}$ which arise in the circumstance of Theorem 0.1.

Let (E, h) be a holomorphic Hermitian vector bundle over a complex manifold M. Let Θ_h denote the curvature form of h. Recall that Θ_h is naturally identified with a Hermitian form along the fibers of $E \otimes T^{1,0}_M$, where $T^{1,0}_M$ denotes the holomorphic tangent bundle of M and that (E, h) is said to be Nakano positive if $\Theta_h > 0$ as such a Hermitian form (cf. [Nk]). If (E, h) is Nakano positive and $\mathrm{rank}E = 1$, this positivity notion is first due to Kodaira [Kd]. For the proof of Theorem 0.1 we shall apply the following generalization of Nakano's vanishing theorem (cf. [Nk]) and Akizuki-Nakano's vanishing theorem (cf. [A-Nk]).

Theorem 2.1. (cf. [A-V]. See also [Kz].) *Let (E, h) be a Nakano positive vector bundle over a complete Kähler manifold (M, g) of dimension n. If $\Theta_h - Id_E \otimes g \geq 0$, then $H^{n,s}_{(2)}(M, E, g, h) = 0$ holds for $s \geq 1$. If* $\mathrm{rank}E = 1$ *and $\Theta_h = g$, one has $H^{r,s}_{(2)}(M, E, g, h) = 0$ for $r + s > n$.*

3. Proof of Theorem 0.1

Let Ω be a weakly pseudoconvex domain of very regular type in a complex manifold M and let (E, h) be a Hermitian holomorphic vector bundle over M whose curvature form Θ_h is Nakano positive. Since each component of $\partial\Omega$ is either a C^2 real hypersurface or a divisor, there exist a function $\psi : M \to [0, \infty)$ of class C^2 with $\psi^{-1}(0) = \partial\Omega$ and a positive number A such that $-\partial\bar{\partial} \log \psi + A\Theta_{\det h} > 0$ holds on Ω, where $\partial\bar{\partial}\rho$ for a real-valued C^2 function ρ is identified with the complex Hessian by an abuse of notation. For such a function ψ, one may take $|s|^2$ for a canonical section s of $[D]$ for any effective divisor D supported on $\partial_{div}\Omega$, on a neighborhood of $\partial_{div}\Omega$, and take ρ^2 for a C^2 defining function ρ of $\partial_{hyp}\Omega$, on a neighborhood of $\partial_{hyp}\Omega$.

If Ω is UBS, ψ can be chosen in such a way that for any $\epsilon > 0$ one can find a neighborhood U of $\partial_{div}\Omega$ such that

$$-\partial\bar{\partial} \log \psi + \epsilon\Theta_{\det h} > 0$$

holds on $U \setminus \partial\Omega$. Moreover we are allowed to modify the metric $-\partial\bar{\partial} \log \psi + \epsilon\Theta_{\det h}$ near $\partial_{div}\Omega$ by adding a term $\partial\bar{\partial}\frac{1}{\log(-\log \psi)}$ so that it becomes complete near $\partial_{div}\Omega$. This can be verified by a straightforward computation.

On the other hand, for any $\epsilon > 0$ one can also find a neighborhood $V \supset \partial_{hyp}\Omega$ such that $-\partial\bar{\partial}\log\psi + \epsilon\Theta_{\det h}$ is a metric on $V \cap \Omega$ which is complete near $\partial_{hyp}\Omega$. This follows immediately from the following lemma.

Lemma 3.1. *Let Ω be a UBS domain in a Hermitian manifold (M, g) and let $\rho : M \to \mathbb{R}$ be a C^3 function satisfying $\overline{\Omega}^{\circ} = \{z; \rho(z) < 0\}$ and $d\rho|_{\partial_{hyp}\Omega} \neq 0$. Then for any $\epsilon > 0$ there exists a neighborhood $U \supset \partial_{hyp}\Omega$ such that $-\rho^{-1}\partial\bar{\partial}\rho + \epsilon(g + \rho^{-2}\partial\rho\bar{\partial}\rho) > 0$ holds on $U \cap \Omega$.*

Proof For simplicity we assume that $\dim M = 2$, since the proof is similar for the general case. Let $x \in \partial_{hyp}\Omega$ and let (z, w) be a local coordinate around x such that the Taylor expansion of ρ at x is given by

$$\rho = \mathrm{Re}w + \rho_2 + \rho_3 + o(3),$$

where $\rho_k = O(k)$.

We put

$$\partial\bar{\partial}\rho_2 = adzd\bar{z} + bdzd\bar{w} + \bar{b}dwd\bar{z} + cdwd\bar{w}.$$

If $a > 0$, it is easy to see that, for any $\epsilon > 0$ one can find a neighborhood $V \ni x$ such that $-\rho^{-1}\partial\bar{\partial}\rho + \epsilon(g + \rho^{-2}\partial\rho\bar{\partial}\rho) > 0$ holds on $V \cap \Omega$.

Let us assume that $a = 0$. Then it follows from $\partial\bar{\partial}\rho(0, 0) \geq 0$ that $b = 0$ and $c \geq 0$. Therefore, by letting

$$\partial\bar{\partial}\rho = Adzd\bar{z} + Bdzd\bar{w} + \bar{B}dwd\bar{z} + Cdwd\bar{w},$$

one sees that $A_z(0, 0) = 0$ and $A_w(0, 0) = 0$ should follow from $\partial\bar{\partial}\rho \geq O(\rho^2)$ on a neighborhood of $\partial_{hyp}\Omega$. Hence, for any $\epsilon > 0$ one can find a neighborhood $V \ni x$ such that $-\rho^{-1}\partial\bar{\partial}\rho + \epsilon(g + \rho^{-2}\partial\rho\bar{\partial}\rho) > 0$ holds on $V \cap \Omega$. Hence, by the compactness of $\partial_{hyp}\Omega$ we have the desired conclusion. □

Consequently we obtain the following, which is crucial for the L^2 estimate needed for the proof of Theorem 0.1.

Lemma 3.2. *If Ω is a UBS domain, one can choose the above ψ in such a way that for any $\epsilon > 0$ there exist a neighborhood $U \supset \partial\Omega$ such that*

$$Id_E \otimes \partial\bar{\partial}\left(-\log\psi + \frac{1}{\log(-\log\psi)}\right) + \epsilon\Theta_h > 0$$

on $U \cap \Omega$.

Proof of Theorem 0.1. Let ψ be as in Lemma 3.2. Then there exists an increasing sequence $(m_\mu) \in \mathbb{R}^{\mathbb{N}}$ such that

$$Id_E \otimes \partial\bar{\partial}\left(-\mu \log \psi + \frac{1}{\log(-\log \psi)}\right) + \Theta_h > 0$$

holds on $\{x \in \Omega; \psi(x) \leq e^{-m_\mu}\}$. We may assume that ψ is C^∞ on Ω.

Therefore, one can find a C^∞ function $\lambda : \mathbb{R} \to \mathbb{R}$ satisfying $\lambda(t) = t$ on $(-1, 1)$, positive numbers a and C, and an increasing sequence of C^∞ convex increasing functions $\lambda_\mu : \mathbb{R} \to \mathbb{R}$ ($\mu \in \mathbb{N}$) such that $\lambda_\mu(t) = 0$ if $t < m_1$, $\lambda_\mu(t) = \mu t$ if $t > m_{\mu+1}$, $\lambda_\mu(t) = \lambda_{\mu+1}(t)$ if $t < m_{\mu+1}$ and

$$Id_E \otimes \left(\partial\bar{\partial}\left(\lambda_\mu(\log \psi) + a\lambda\left(\frac{1}{\log(-\log \psi + C)}\right)\right)\right) + \Theta_h > 0$$

on Ω.

Hence, for every $\mu \in \mathbb{N}$ one can find positive numbers ϵ_μ and δ_μ such that

$$g_{\epsilon_\mu,\delta_\mu} := \epsilon_\mu \partial\bar{\partial}\left(\lambda_\mu(\log \psi) + a\lambda\left(\frac{1}{\log(-\log \psi + C)}\right)\right) + \delta_\mu \Theta_{\det h}$$

is a complete Kähler metric on Ω satisfying

$$Id_E \otimes \left(\partial\bar{\partial}\left(\lambda_\mu(\log \psi) + a\lambda\left(\frac{1}{\log(-\log \psi + C)}\right)\right)\right) + \Theta_h > Id_E \otimes g_{\epsilon_\mu,\delta_\mu}.$$

Therefore, by Theorem 2.1 we obtain

$$H^{n,s}_{(2)}(\Omega, E, g_{\epsilon_\mu,\delta_\mu}, h\psi^{-\mu}) (\cong H^{n,s}_{(2)}(\Omega, E, g_{\epsilon_\mu,\delta_\mu}, he^{-\lambda_\mu(\log \psi)})) = 0 \text{ for } s \geq 1,$$

since $\lambda(\frac{1}{\log(-\log \psi + C)})$ is bounded.

Now let $\mu \in \mathbb{N}$ and let v be any representative of an element of $H^{n,s}_{(2)}(\Omega, E, \Theta_{\det h, h\delta^\mu})$ ($s \geq 1$). Then it is clear that one can find $\nu \geq \mu$ such that $v \in L^{n,s}_{(2)}(\Omega, E, g_{\epsilon_\nu,\delta_\nu}, h\psi^{-\nu})$, so that by the above vanishing of $H^{n,s}_{(2)}(\Omega, E, g_{\epsilon_\nu,\delta_\nu}, h\psi^{-\nu})$ $\bar{\partial}u = v$ holds for some $u \in L^{n,s-1}_{(2)}(\Omega, E, g_{\epsilon_\nu,\delta_\nu}, h\psi^{-\nu})$. Since

$$L^{r,s}_{(2)}(\Omega, E, g_{\epsilon_\nu,\delta_\nu}, h\psi^{-\nu}) \subset \bigcup_{\kappa=1}^{\infty} L^{r,s}_{(2)}(\Omega, E, \Theta_{\det h}, h\delta^\kappa)$$

it follows that v represents zero in $H^{n,s}_{p.g}(\Omega, E)$.

Similarly one has $H^{r,s}_{p.g.}(\Omega, E) = 0$ if rank$E = 1$ and $r + s > n$. □

Remark 3.1. If a complex manifold M is mapped onto a Stein space V by a holomorphic map f and (E, h) is a Nakano positive Hermitian holomorphic vector bundle over M, Theorem 0.1 can be generalized to a vanishing theorem on a locally pseudoconvex domain $\Omega \subset M$ such that $\partial\Omega$ consists of real hypersurfaces and divisors in such a way that the restriction of f to them is proper, where the UBS condition is imposed similarly as in the case of bounded domains. As a corollary, one has the corresponding vanishing for the direct images of relatively algebraic sheaves. In case Ω is a smooth family over V with respect to f equipped with a divisor D for which $f|_D$ is proper, it may be an interesting question to extend the theorems in [L-R-W] and [L-W-Y] to this situation.

References

[A-Nk] Akizuki, Y. and Nakano, S., *Note on Kodaira-Spencer's proof of Lefschetz theorems,* Proc. Japan Acad. **30** (1954), 266-272

[A-V] Andreotti, A. and Vesentini, E., *Sopra un teorema di Kodaira*, Ann. Scuola Norm. Sup. Pisa **15** (1961), 283-309.

[B-S] Boas, H. P. and Straube, E. J., *Sobolev estimates for the $\bar{\partial}$-Neumann operator on domains in $\mathbb{C}^n$ admitting a defining function that is plurisubharmonic on the boundary,* Math. Z. **206** (1991), no. 1, 81-88.

[D-1] Deligne, P. *Théorème de Lefschetz et critères de dégénérescence de suites spectrales,* Inst. Hautes Études Sci. Publ. Math. **35** (1968), 259-278.

[D-2] ——, *Théorie de Hodge II,* Publ. Math. Inst. Hautes Études Sci. Publ. Math. **40** (1972), 5-57.

[H] Hörmander, L., *L^2 estimates and existence theorems for the $\bar{\partial}$-operator,* Acta Math.**113** (1965), 89-152.

[Kz] Kazama, H., *Approximation theorem and application to Nakano's vanishing theorem for weakly 1-complete manifolds*, Mem. Fac. Sci. Kyushu Univ. **27** (1973), 221-240.

[Kd] Kodaira, K. *On a differential-geometric method in the theory of analytic stacks,* Proc. Nat. Acad. Sci. U. S. A. **39**, (1953). 1268-1273.

[K] Koike, T., *Linearlization of transition functions of a semi-positive line bundle along a certain submanifold,* arXiv:2002.07830 *to appear in Ann. Inst. Fourier.*

[L-R-W] Liu, K., Rao, S. and Wan, X., *Geometry of logarithmic forms and deformations of complex structures,* J. Algebraic Geom. **28** (2019), no. 4, 773-815.

[L-W-Y] Liu, K.-F., Wan, X.-Y. and Yang, X.-K., *Logarithmic vanishing theorems for effective q-ample divisors,* Sci. China Math. **62** (2019), no. 11, 2331-2334.

[M] Maltsiniotis, G., *G.A.G.A. affine (d'après Pierre Deligne)* Différents aspects de la positivité, Sém. Géom. Anal., École Norm. Sup., l972-1973, pp. 141-160. Astérisque, No. 17, Soc. Math. France, Paris, 1974.

[Nk] Nakano, S., *On complex analytic vector bundles*, J. Math. Soc. Japan **7** (1955), 1-12.

[Nk-R] Nakano, S. and Rhai, T.-S., *Vector bundle version of Ohsawa's finiteness theorems,* Math. Japon. **24** (1979/80), no. 6, 657-664.

[N] Norimatsu, Y., *Kodaira vanishing theorem and Chern classes for ∂-manifolds,* Proc. Japan Acad. Ser. A Math. Sci. **54** (1978), no. 4, 107-108.

[Oh-1] Ohsawa, T., *Finiteness theorems on weakly 1-complete manifolds,* Publ. Res. Inst. Math. Sci. **15** (1979), no. 3, 853-870.

[Oh-2] ——, *A remark on Hörmander's isomorphism,* Complex analysis and geometry, 273-280, Springer Proc. Math. Stat., 144, Springer, Tokyo, 2015.

[Oh-3] ——, *Variants of Hörmander's theorem on q-convex manifolds by a technique of infinitely many weights,* Abhandlungen aus dem mathematischen Seminar der Universität Hamburg 2021 (online).

[Oh-4] ——, *L^2 $\bar{\partial}$-cohomology with weights and bundle convexity of certain locally pseudoconvex domains,* preprint.

[Oh-5] ——, *On weakly pseudoconvex domains of regular type – an approximation theorem,* preprint.

[S] Sasakura, N., *Cohomology with polynomial growth and completion theory,* Publ. Res. Inst. Math. Sci. **17** (1981), no. 2, 371-552.

[U] Ueda, T., *On the neighborhood of a compact complex curve with topologically trivial normal bundle,* J. Math. Kyoto Univ. **22** (1982/83), no. 4, 583—607.

Takeo Ohsawa, Graduate School of Mathematics Nagoya University 464-8602 Chikusaku Furocho Nagoya Japan

Email address: `ohsawa@math.nagoya-u.ac.jp`