

The 28th Symposium of Complex Geometry (Kanazawa) 2022

	November 7
15:00-15:40	Miyatake
16:00-16:40	Oono

	November 8	November 9	November 10	November 11
9:15-9:30	tea	tea	tea	tea
9:30-10:30	Ohsawa	Takayama	Yoshikawa	Mabuchi
10:50-11:50	R. Kobayashi*	Nakagawa	Tsuji	Noguchi
Lunch				
13:45-14:00	tea	tea	tea	
14:00-15:00	Kasuya	Futaki*	Iwai	
15:20-16:20	Kawakami	Inoue	Imaike	
16:40-17:40	Takahashi*	Hashimoto	Aoi	

Venue: Shinoki geihnkkan (しいのき迎賓館) 金沢

(*) indicates online speakers.

Nov. 7

15:00-15:40 Natsuo Miyatake (Kyushu University)

Title: Uniformization of compact Sasakian manifolds using basic Higgs bundles

16:00-16:40 大野 高志 (Osaka University)

Title: Deformations of Higgs Bundles

Nov. 8

9:30- 10:30 Takeo Ohsawa (Nagoya University)

Title: Bundle-convexity of locally pseudoconvex domains

10:50-11:50 Ryoichi Kobayashi (Nagoya University)

Title: Construction of equi-dimensional full-rank holomorphic maps via measure theoretic analysis of position and its application to problems in complex geometry

14:00-15:00 Hisashi Kasuya (Osaka University)

Title: Geometry of double sided actions on Lie groups, new examples in Kähler and non-Kähler geometry

15:20-16:20 Yu Kawakami (Kanazawa University)

Title: Recent development in value distribution theory of the Gauss map of complete minimal surfaces

16:40-17:40 Ryosuke Takahashi (Kyushu University)

Title: A Nakai—Moishezon type criterion for supercritical deformed Hermitian—Yang—Mills equation

Nov. 9

9:30- 10:30 Shigeharu Takayama (The University of Tokyo)

Title: 多重標準線形系の L^p 構造の極限について

10:50-11:50 Yasuhiro Nakagawa (Kumamoto University)

Title: Multiplier Hermitian-Einstein metrics on KSM-manifolds

14:00-15:00 Akito Futaki (Tsinghua University, The University of Tokyo)

Title: Moment polytopes on Sasaki manifolds and volume minimization

15:20-16:20 Eiji Inoue (RIKEN)

Title: A sketch of the future of μ -cscK metrics and μK -stability

16:40-17:40 Yoshinori Hashimoto (Osaka Metropolitan University)

Title: Coupled Ding stability and related topics

Nov. 10

9:30- 10:30 Ken'ichi Yoshikawa (Kyoto University)

Title: Degeneration of Riemann surfaces and small eigenvalues of Laplacian

10:50-11:50 Hajime Tsuji (Sophia University)

Title: Application of p -Bergman kernels

14:00-15:00 Masataka Iwai (Osaka University)

Title: Abundance theorem for minimal compact Kähler manifolds with vanishing second Chern class

15:20-16:20 Dai Imai (Kyoto University)

Title: Analytic torsion for irreducible holomorphic symplectic fourfolds with antisymplectic involution

16:40-17:40 青井顕宏 (阿武野高等学校)

Title: A conical approximation of constant scalar curvature Kähler metrics of Poincaré type and log K-semistability

Nov. 11

9:30- 10:30 Toshiki Mabuchi (Osaka University)

Title: The degeneracy of the Hodge spectral sequence for manifolds of class C; a generalization of Fujiki's idea

10:50-11:50 Junjiro Noguchi (The University of Tokyo)

Title: Analytic Ax-Schanuel Theorem for semi-abelian varieties and Nevanlinna theory

Natsuo Miyatake (Kyushu University)

Title: Uniformization of compact Sasakian manifolds using basic Higgs bundles

abstract: Simpson introduced the notion of uniformizing variation of Hodge structure on compact Kähler manifolds and showed that the universal covering of a compact Kähler manifold is isomorphic to a bounded symmetric domain if and only if it exists on the manifold. He also gave a characterization of Higgs bundles which give the uniformizing variation of Hodge structure. In this talk, on compact Sasakian manifolds, we consider an analogy of this. We can formulate an analogy of the uniformization variation of Hodge structure also on compact Sasakian manifolds. However, even if it exists, the constructed map collapses all the points on each leaf of the foliation of the universal covering into one point and can never be an isomorphism. I will explain how to properly take the lift of this map and construct an isomorphism from the universal covering to a homogeneous space. This is joint work with Hisashi Kasuya.

大野 高志 (Osaka University)

Title: Deformations of Higgs Bundles

abstract: The purpose of this talk is to introduce the DGLA which governs the deformations of Higgs bundles. Since we construct the DGLA via differential geometric approach, the differential of this DGLA has an explicit form which we could not obtain previously. Using the differential we obtained here, we can construct the Kuranishi Space of Higgs bundles.

Takeo Ohsawa (Nagoya University)

Title: Bundle-convexity of locally pseudoconvex domains

abstract: A complex manifold is said to be complete Kähler at infinity if it admits a complete Hermitian metric whose fundamental form is closed outside a compact subset. A large amount of locally pseudoconvex domains have this property. It is expected that basic existence theorems on strongly pseudoconvex manifolds for coherent analytic sheaves have natural generalizations on such manifolds for locally free sheaves under appropriate positivity conditions on the curvature. Affirmative results in this direction will be shown.

Ryoichi Kobayashi (Nagoya University)

Title: Construction of equi-dimensional full-rank holomorphic maps via measure theoretic analysis of position and its application to problems in complex geometry

abstract: We propose existence results of equidimensional maximal rank holomorphic maps from \mathbb{C}^n to complex projective varieties of dimension n . The proposed construction is interpreted as a higher dimensional generalization of the construction of an infinite cyclic covering map from \mathbb{C}^* to an elliptic curve based on the existence of self-coverings of an elliptic curve. What makes this generalization possible is the effect of the measure concentration phenomenon to the analysis of position (i.e., measure theoretic intersection theory). We then discuss applications of our construction to various problems in complex geometry.

Hisashi Kasuya (Osaka University)

Title: Geometry of double sided actions on Lie groups, new examples in Kähler and non-Kähler geometry

abstract: It is well-known that even dimensional compact Lie groups admit left-invariant complex structures. Recent studies suggest that there richly exist complex structures on even dimensional compact Lie groups which are different from such invariant complex structures. By using double sided actions on Lie groups, we give a way of constructing explicit "non-invariant" complex structures. Consequently we obtain orbifold (foliation) variants of flag varieties. This talk is based on the joint work with Hiroaki Ishida (Kagoshima).

Yu Kawakami (Kanazawa University)

Title: Recent development in value distribution theory of the Gauss map of complete minimal surfaces

abstract: In this talk, we give an overview of recent works on value distribution theory of the Gauss map of complete minimal surfaces in Euclidean space. This is based on a joint work with Pham Hoang Ha and Mototsugu Watanabe.

Ryosuke Takahashi (Kyushu University)

Title: A Nakai—Moishezon type criterion for supercritical deformed Hermitian—Yang—Mills equation

abstract: In this talk, we show that the solvability of the supercritical deformed Hermitian—Yang—Mills equation is equivalent to a certain algebraic positivity condition modeled on the Nakai—Moishezon ampleness criterion, which confirms the mirror version of the Thomas—Yau conjecture. This is a joint work with J. Chu (Peking University) and M.-C. Lee (Chinese University of Hong Kong).

Shigeharu Takayama (The University of Tokyo)

Title: 多重標準線形系の L^p 構造の極限について

abstract: 多重標準線形系には標準的な L^p 空間の構造が入る。代数多様体の 1 パラメータ族の退化において、一般ファイバーの L^p 構造がある混合 L^p 構造に収束することを示す。

Yasuhiro Nakagawa (Kumamoto University)

Title: Multiplier Hermitian-Einstein metrics on KSM-manifolds

abstract: In this talk, we focus on multiplier Hermitian-Einstein metrics introduced by Mabuchi, which include Kähler-Einstein metrics, Kähler-Ricci solitons and Mabuchi solitons as special cases. We establish a criterion for the existence of multiplier Hermitian-Einstein metrics for a KSM-manifold, which has the structure of a fiber bundle over an Einstein-Kähler Fano manifold whose fiber is a toric Fano manifold. This talk is based on joint works with Satoshi Nakamura.

Akito Futaki (Tsinghua University, The University of Tokyo)

Title: Moment polytopes on Sasaki manifolds and volume minimization

abstract: We show that transverse coupled Kähler-Einstein metrics on toric Sasaki manifolds arise as a critical point of a volume functional. As a preparation for the proof, we re-visit the transverse moment polytopes and contact moment polytopes under the change of Reeb vector fields. Then we apply it to a coupled version of the volume minimization by

Martelli-Sparks-Yau. This is done assuming the Calabi-Yau condition of the Kähler cone, and the non-coupled case leads to a known existence result of a transverse Kähler-Einstein metric and a Sasaki-Einstein metric, but the coupled case requires an assumption related to Minkowski sum to obtain transverse coupled Kähler-Einstein metrics.

Eiji Inoue (RIKEN)

Title: A sketch of the future of μ -cscK metrics and μ K-stability

abstract: There are various notions of canonical metric on Kahler manifold; Kahler-Einstein metric, cscK metric, extremal metric, Kahler-Ricci soliton and so on, all of which carry corresponding algebro-geometric stability notions of polarized variety characterizing the existence of canonical metrics. For the last three years, I have studied a framework which encloses all of the above notions of canonical metric and K-stability, which I call μ -framework of canonical metrics and K-stability. As I briefly explained the current structure of the theory in the last two talks of this symposium, this time, I would rather illustrate the theory with a few examples and share fundamental open problems to sketch the future program.

... I should remind there are still many different notions of canonical metrics and K-stability which are not covered in the μ -framework. The moment map picture and the YTD conjecture for μ -cscK metrics is a fundamental aspect, but the unique feature of the μ -framework is rather the minimax picture on Perelman's entropy, optimal destabilization conjecture and a phase transition phenomenon in volume minimization, which I will focus on in the talk.

Yoshinori Hashimoto (Osaka Metropolitan University)

Title: Coupled Ding stability and related topics

abstract: Hultgren and Witt Nyström introduced coupled Kähler-Einstein metrics, which are defined for a tuple of Kähler metrics whose Kähler classes add to the anticanonical class, and generalise Kähler-Einstein metrics. They also proposed a coupled version of the Yau-Tian-Donaldson conjecture, stating that the existence of coupled Kähler-Einstein metrics is equivalent to the coupled Ding stability, and proved one direction of the conjecture. We strengthen their definition of coupled Ding stability so that the connection with differential geometry (particularly the variational principle) is more explicit. We show that the existence of coupled Kähler-Einstein metrics implies this strengthened version of coupled Ding semistability, and present results related to the balanced metrics. We also mention various open problems concerning this strengthened version of coupled Ding stability.

Ken'ichi Yoshikawa (Kyoto University)

Title: Degeneration of Riemann surfaces and small eigenvalues of Laplacian

abstract: Let $f : X \rightarrow S$ be a one parameter family of compact Riemann surfaces over a curve. Assume that X is a compact Kähler surface. Then the fibers of f are endowed with the metric induced from the Kähler metric on X . If the singular fiber of f is not irreducible, then some eigenvalues of the Laplacian of the regular fiber converge to zero as the regular fiber approaches the singular fiber. We call such eigenvalues small eigenvalues. Here the Laplacian means the one acting on the functions of each regular fiber. In this talk, when the singular fiber is reduced, I explain the asymptotic behavior of the "product" of all small eigenvalues of Laplacian. This is a joint work with Xianzhe Dai.

Hajime Tsuji (Sophia University)

Title: Application of p -Bergman kernels

abstract: In this talk, I would like to present some applications of p -Bergman kernels for (1) $p < 1$ and (2) $p = \infty$.

For $p < 1$, the variation of p -Bergman kernel is known to be log-psh for projective deformation. We consider the application of this fact to the deformation of Kähler varieties with nonnegative Kodaira dimension. For $p = \infty$, we shall prove the Banach space of bounded holomorphic functions Determines a bounded strongly pseudoconvex domains in \mathbf{C}^n .

Masataka Iwai (Osaka University)

Title: Abundance theorem for minimal compact Kähler manifolds with vanishing second Chern class

abstract: In this talk, I will discuss the abundance conjecture, which states that if the canonical line bundle is nef, then the canonical line bundle is semiample. I will talk about the abundance theorem and structure theorem for compact Kähler manifolds with nef canonical line bundles and vanishing second Chern class. This is joint work with Shin-ichi Matsumura.

Dai Imai (Kyoto University)

Title: Analytic torsion for irreducible holomorphic symplectic fourfolds with antisymplectic involution

abstract: In 2004, Yoshikawa constructed an invariant of 2-elementary K3 surfaces by

using analytic torsion. He also proved that the invariant is expressed as the Petersson norm of a certain automorphic form on a bounded symmetric domain of type IV and a certain Siegel modular form. In this talk, we generalize this result to a class of higher dimensional manifolds. More precisely, we construct an invariant of irreducible holomorphic symplectic manifolds which are deformation equivalent to the Hilbert scheme of a K3 surface with antisymplectic involution by using the analytic torsion. We show that the invariant is expressed as the Petersson norm of a certain automorphic form on a bounded symmetric domain of type IV and a certain Siegel modular form in some cases.

青井顕宏 (阿武野高等学校)

Title: A conical approximation of constant scalar curvature Kähler metrics of Poincaré type and log K-semistability

abstract: Guenancia proved that a Kähler-Einstein metric of Poincaré type is the limit of a family of Kähler-Einstein metrics with cone singularities along a smooth divisor. In this talk, I will explain the recent result which is a constant scalar curvature Kähler version of Guenancia's result. In addition, I will explain that the existence of constant scalar curvature Kähler metrics of Poincaré type implies log K-semistability with angle 0.

Toshiki Mabuchi (Osaka University)

Title: The degeneracy of the Hodge spectral sequence for manifolds of class C; a generalization of Fujiki's idea

abstract: By Fujiki, the Hodge spectral sequence degenerates for manifolds of class C. In this talk, his method will be studied in a more general situation. An application will also be given.

Junjiro Noguchi (The University of Tokyo)

Title: Analytic Ax-Schanuel Theorem for semi-abelian varieties and Nevanlinna theory

abstract: The present study is motivated by *Schanuel Conjecture*, which in particular implies the algebraic independence of e and π . Our aim is to explore, as a transcendental functional analogue of Schanuel Conjecture, the value distribution theory (Nevanlinna theory) of the entire curve $\widehat{\exp}_A f := (\exp_A f, f) : \mathbf{C} \rightarrow A \times \text{Lie}(A)$ associated with an entire curve $f : \mathbf{C} \rightarrow \text{Lie}(A)$, where $\exp_A : \text{Lie}(A) \rightarrow A$ is an exponential map of a semi-abelian variety A .

We firstly give a Nevanlinna theoretic proof to the *analytic Ax-Schanuel Theorem* for semi-abelian varieties, which was proved by J. Ax 1972 in the case of formal power series

$\mathbf{C}[[t]]$ (Ax-Schanuel Theorem). We assume some non-degeneracy condition for f such that in the case of $A = (\mathbf{C}^*)^n$ and $\text{Lie}((\mathbf{C}^*)^n) = \mathbf{C}^n$, the elements of the vector-valued function $f(z) - f(0)$ are \mathbf{Q} -linearly independent. Then by the method of Nevanlinna theory (the Log Bloch-Ochiai Theorem), we prove that $\text{tr.deg}_{\mathbf{C}} \widehat{\text{ex}}_A f \geq n + 1$.

Secondly, we prove a *Second Main Theorem* for $\widehat{\text{exp}}_A f$ and an algebraic divisor D on $A \times \text{Lie}(A)$ with compactifications $\bar{D} \subset \bar{A} \times \overline{\text{Lie}(A)}$ such that

$$T_{\widehat{\text{exp}}_A f}(r, L(\bar{D})) \leq N_1(r, (\widehat{\text{exp}}_A f)^* D) + \varepsilon T_{\text{exp}_A f}(r) + O(\log r) \quad ||_{\varepsilon}.$$

We will also deal with the intersections of $\widehat{\text{ex}}_A f$ with higher codimensional algebraic cycles of $A \times \text{Lie}(A)$ as well as the case of higher jets.

We will apply the result to a uniqueness problem for holomorphic curves of type $\widehat{\text{exp}}_A f$. We will discuss the case when the domain is a punctured disk, affine algebraic curves, and other open Riemann surfaces.