# The 29th Symposium of Complex Geometry (Kanazawa) 2023

	December 4
15:00-15:40	Miyatake
16:00-16:40	Kasao

	December 5	December 6	December 7	December 8
9:15-9:30	tea	tea	tea	tea
9:30-10:30	Ohsawa	Odaka	Fujita	Noguchi
10:50-11:50	Odaka	Xia Mingchen	Tsuji	Yamanoi
Lunch				Mabuchi
				(12:10-13:10)
13:45-14:00	tea	tea	tea	
14:00-15:00	R. Kobayashi	Keita Goto	Hattori	
15:20-16:20	Iwai	Keizo	Miyachi	
		Hasegawa		
16:40-17:40	Inayama	Kazuyuki	Kikuta	
		Hasegawa		

Venue: Shinoki geihnkan (しいのき迎賓館) 金沢

## Dec. 4

15:00-15:40 Natsuo Miyatake (Tohoku University) Title: Extended Hitchin equation for cyclic Higgs bundles associated with a quasi-subharmonic function, and its Dirichlet problem

16:00-16:40 Shunsuke Kasao (Kanazawa University) Title: The Bloch-Ros principle and its application

## Dec. 5

9:30- 10:30 Takeo Ohsawa (Nagoya University) Title: On Coeure-Loeb's counterexample to the Serre problem 10:50-11:50 Yuji Odaka (Kyoto University) Title: A survey on developments around K-stability I, II

14:00-15:00 Ryoichi Kobayashi (Nagoya University) Title: Measure Theoretic Approach to Holomorphic Maps

15:20-16:20 Masataka Iwai (Osaka University) Title: Positivity of tangent sheaves of projective klt varieties

16:40-17:40 Takahiro Inayama (Tokyo University of Science) Title: Towards singular Nakano positivity

## Dec. 6

9:30- 10:30 Yuji Odaka (Kyoto University)Title: A survey on developments around K-stability I, II10:50-11:50 Xia Mingchen (IMJ-PRG)Title: Singularities in global pluripotential theory

14:00-15:00 Keita Goto (National Taiwan University) Title: On Non-Archimedean Analog of SYZ Fibration

15:20-16:20 Keizo Hasegawa (Osaka University/Niigata University) Title: Cartan Flat Non-degenerate CR Lie Groups

16:40-17:40 Kazuyuki Hasegawa (Kanazawa University) Title: The H/Q-correspondence and a generalization of the supergravity c-map

# Dec. 7

9:30- 10:30 Kento Fujita (Osaka University)
Title: K-stability of Casagrande–Druel varieties
10:50-11:50 Hajime Tsuji (Sophia University)
Title: The Limits of Kähler-Ricci flows
14:00-15:00 Kota Hattori (Keio University)
Title: Smooth maps minimizing the energy and the calibrated geometry
15:20-16:20 Hideki Miyachi (Kanazawa University)
Title: Complex analytical aspects in Teichmüller theory
16:40-17:40 Kohei Kikuta (Osaka University)

Title: Geometrically finiteness for automorphism groups of K3 surfaces

#### Dec. 8

9:30-10:30 Junjiro Noguchi (The University of Tokyo)

Title: A geometric bound of the number of rational points in Manin–Grauert's Theorem

10:50-11:50 Katsutoshi Yamanoi (Osaka University)

Title: Bloch's principle for holomorphic maps into subvarieties of semi-abelian varieties

12:10-13:10 Toshiki Mabuchi (Osaka University)

Title: Kodaira dimension for Kaehlerian deformation

#### Abstracts

#### Dec. 4

Natsuo Miyatake (Tohoku University)

Title: Extended Hitchin equation for cyclic Higgs bundles associated with a quasi-subharmonic function, and its Dirichlet problem

**abstract:** Let X be a Riemann surface endowed with the canonical bundle  $K_X \to X$ . We select a square root  $K_X^{1/2}$  of the canonical bundle. Using this, we define a holomorphic vector bundle E of rank r as  $E := \bigoplus_{j=1}^r K_X^{(r-(2j-1))/2}$ . For every  $q \in H^0(K_X^r)$ , we naturally associate an EndE-valued holomorphic 1-form, denoted by  $\Phi(q) \in H^0(\text{End}E \otimes K_X)$ . This pair  $(E, \Phi(q))$  is called a cyclic Higgs bundle. From  $\Phi(q)$ , we derive a second-order elliptic PDE for a diagonal Hermitian metric  $h = (h_1, \ldots, h_r)$  on E. This is known as the Hitchin equation for the cyclic Higgs bundle  $(E, \Phi(q))$ . The solution of the Hitchin equation for a cyclic Higgs bundle yields a harmonic map from the universal covering space to the symmetric space  $SL(r, \mathbb{C})/SU(r)$ . In this talk, we will extend the cyclic Higgs bundle concept to be associated with a multi-valued r-differential,  $q_N^{1/N}$ , where  $q_N \in H^0((K_X^r)^N)$ . We also pose the problem of considering the behavior of the solution of the Hitchin equation and various quantities derived from the solution as N approaches infinity. Motivated by this, we introduce a generalization of the Hitchin equation for a cyclic Higgs bundle associated with a quasi-subharmonic function  $\varphi$ . Our main result is the existence and uniqueness of the solution to the Dirichlet problem for this generalized equation.

Shunsuke Kasao (Kanazawa University)

Title: The Bloch-Ros principle and its application

**abstract:** We give an overview of the relationship among the Picard theorem (in value distribution), the Montel theorem (which is famous as the criterion for normal families) and the Fujimoto theorem (in minimal surface). Moreover, we give the applications of this relationship to the Gauss map of several classes of surfaces. This is based on a joint work with Yu Kawakami.

#### Dec. 5

Takeo Ohsawa (Nagoya University)

Title: On Coeure-Loeb's counterexample to the Serre problem

**abstract:** It was remarked by Diederich and Coltoiu in 2007 that Coeure-Loeb's counterexample (=CLE) to the Serre problem gives a negative answer to a question asked by Griffiths in 1977. After reviewing this, function-theoretic properties of CLE will be discussed from the  $L^2$  viewpoint. It is planned to extend a result on bounded locally pseudoconvex domains whose canonical bundle is negative on a neighborhood of the boundary.

Yuji Odaka (Kyoto University)

Title: A survey on developments around K-stability I, II

**abstract:** I will try a survey talk on recent developments around K-stability, from what I understand. The latter half will involve some discussions towards algebro-geometric understanding of open/complete Ricci-flat Kahler metrics, hopefully.

Ryoichi Kobayashi (Nagoya University)

Title: Measure Theoretic Approach to Holomorphic Maps

**abstract:** In this lecture I introduce random projection of projective varieties. The large dimension of the ambient projective space gives rise to the measure concentration phenomenon to the related linear algebraic objects, e.g., the space of random linear projections. I will propose applications of this principle to holomorphic curves in a projective variety.

Masataka Iwai (Osaka University)

Title: Positivity of tangent sheaves of projective klt varieties

**abstract:** In this talk, we will introduce the structure theorem of projective klt varieties with certain positive tangent sheaves. More precisely, if the tangent sheaf is almost nef or positively curved, after taking a quasi-etale cover, we can take a well-defined MRC fibration onto an Abelian variety. Furthermore, we will introduce how the tangent sheaf is related to the structure of projective klt varieties. This is based on joint work with Shin-ichi Matsumura (Tohoku University) and Guolei Zhong (IBS-CCG) (arXiv:2309.09489).

Takahiro Inayama (Tokyo University of Science)

Title: Towards singular Nakano positivity

**abstract:** We discuss various methods for defining Nakano positivity for singular Hermitian metrics. As applications, we provide examples of direct image sheaves with Nakano positive singular Hermitian metrics and show vanishing theorems. This is a joint work with Shin-ichi Matsumura.

#### Dec. 6

Xia Mingchen (IMJ-PRG)

Title: Singularities in global pluripotential theory

**abstract:** In the study of the geometry of complex projective manifolds or compact Kähler manifolds, it is important to understand a class of functions on these manifolds, called quasi-plurisubharmonic functions. Traditionally, one is interested in regular or mildly singular quasi-plurisubharmonic functions as the solutions to the Monge—Ampere equations are usually regular. In recent years, it is gradually realized that singular quasiplurisubharmonic functions also play an important role. In this talk, I will give a general introduction to the systematic study of the singularities of quasi-plurisubharmonic functions carried out by Darvas, Di Nezza, Lu and me in the last few years. In particular, I will explain the important notion of I-good singularities and its various applications.

Keita Goto (National Taiwan University)

Title: On Non-Archimedean Analog of SYZ Fibration

**abstract:** In the SYZ mirror symmetry context, SYZ fibrations has been studied for many years. In 2006, Kontsevich and Soibelman introduce a non-Archimedean analog of SYZ fibration which is called a non-Archimedean SYZ fibration later. Moreover, they predict a certain equivalence between SYZ fibrations and non-Archimedean SYZ fibrations for maximally degenerating families of polarized Calabi-Yau varieties. This talk will be devoted to describing my joint work (https://doi.org/10.1093/imrn/rnad052) with Yuji Odaka, which partially proves this conjecture.

#### Keizo Hasegawa (Osaka University/Niigata University)

#### Title: Cartan Flat Non-degenerate CR Lie Groups

**abstract:** In this talk we discuss non-degenerate CR structures on Lie groups (algebras). We see that a nilpotent Lie algebra admits non-degenerate CR structures if and only if it is Heisenberg Lie algebra. We also see as a main result that the only Cartan flat non-degenerate CR Lie algebras are su(2), sl(2, R),  $Aff(R) \ge R$  and h(2m+1) with its modifications, where Aff(R) is the affine Lie algebra of dimension 2 and h(2m+1) is the Heisenberg Lie algebra of dimension 2m+1. Furthermore, we determine all the (flat and non-flat) non-degenerate CR structures on each of these Lie algebras. This is a joint work with Hisashi Kasuya.

#### Kazuyuki Hasegawa (Kanazawa University)

Title: The H/Q-correspondence and a generalization of the supergravity c-map Given a hypercomplex manifold M with a rotating vector field and addiabstract: tional data, we construct a conical hypercomplex manifold  $\hat{M}$  of dim  $\hat{M} = \dim M + 4$ with  $\mathbb{H}^* (:= \mathbb{H} \setminus \{0\})$ -action and show that  $\overline{M} := \widehat{M} / \mathbb{H}^*$  possesses a quaternionic structure. As a consequence, we associate the quaternionic manifold  $\overline{M}$  to the hypercomplex manifold M of the same dimension. This is a generalization of the HK/QK-correspondence. Here HK and QK mean hyperKähler and quaternionic Kähler, respectively. We call this construction  $\overline{M}$  from M the H/Q-correspondence. We can apply the H/Q-correspondence to a hypercomplex Hopf manifold which does not admit any hyperKähler structure. A compact Lie group SU(3) with the left invariant hypercomplex structure can be also applied to the H/Q-correspondence. These examples show that the H/Q-correspondence is a proper generalization of HK/QK-correspondence. As an application, we show that a quaternionic manifold can be associated to a conical special complex manifold of half its dimension. Furthermore, a projective special complex manifold (with a canonical c-projective structure) associates with a quaternionic manifold. The latter is a generalization of the supergravity c-map. This is a joint work with Vicente Cortés.

#### Dec. 7

Kento Fujita (Osaka University) Title: K-stability of Casagrande–Druel varieties **abstract:** We focus on a certain subclass of Fano varieties named Casagrande—Druel varieties. Especially, we see the K-polystability of several Casagrande—Druel threefolds whose general members are in Mori–Mukai's list Nos. 3.9 and 4.2, and see the K-moduli spaces parametrizing those varieties. This is a joint work with Ivan Cheltsov, Tiago Duarte Guerreiro, Igor Krylov and Jesus Martinez Garcia.

### Hajime Tsuji (Sophia University)

Title: The Limits of Kähler-Ricci flows

**abstract:** In this talk, I will present a proof that on a smooth projective variety with abundant canonical bundle the normalized limit of a Kähler-Ricci flow is the pullback of the twisted Kähler-Einstein form on the base of the Iitaka fibration.

Kota Hattori (Keio University)

Title: Smooth maps minimizing the energy and the calibrated geometry

**abstract:** We generalize the notion of calibrated submanifolds to smooth maps and show several examples of smooth maps that fit our situation. Moreover, we give the energy functional which is minimized by the identity maps on the Riemannian manifolds with special holonomy groups.

## Hideki Miyachi (Kanazawa University)

Title: Holomorphic functions on Teichmuller space

**abstract:** In this talk, we review recent progress on (speaker's) research on the complex analytical aspect in Teichmuller theory. After recalling basic facts in Teichmuller theory, we will review the Poisson integral formula for pluriharmonic functions on Teichmuller space, and discuss the bounded holomorphic functions on Teichmuller space. If time permits, we will discuss problems on our research.

#### Kohei Kikuta (Osaka University)

Title: Geometrically finiteness for automorphism groups of K3 surfaces

**abstract:** The automorphism groups of K3 surfaces have been studied for a long time, but geometric group theoretic research in this area are quite limited. This group naturally acts isometrically (with finite kernel and discrete image) on the classical hyperbolic space. Therefore, understanding this action is important when investigating automorphism groups. The geometrically finiteness is a fundamental property of discrete subgroups of isometries of the hyperbolic spaces. In this talk, we introduce the current understanding of the geometrically finiteness of automorphism groups of K3 surfaces. If time permits, we would also like to discuss rank-1 isometries of the truncated convex hulls of limit sets, and their relationship with the topological entropy.

#### Dec. 8

Junjiro Noguchi (The University of Tokyo)

Title: A geometric bound of the number of rational points in Manin–Grauert's Theorem **abstract:** The finiteness of the rational points of an algebraic curve with higher genus over a function field is known as the Manin ('63)–Grauert ('65) Theorem. In this talk we discuss the proofs of the Manin–Grauert Theorem, and show a bounds of the number of the rational points based on the geometry of the associated fiber space of curves over a compact Riemann surface.

Katsutoshi Yamanoi (Osaka University)

Title: Bloch's principle for holomorphic maps into subvarieties of semi-abelian varieties **abstract:** We discuss a generalization of Bloch-Ochiai-Kawamata-Noguchi's theorem about entire curves in subvarieties X of semi-abelian varieties to the situation of the sequences of holomorphic maps from the unit disc into X. This generalization implies that subvarieties of log general type in semi-abelian varieties are pseudo-Kobayashi hyperbolic. Another application is an improvement of a classical theorem due to Cartan in 1920's about the system of nowhere vanishing holomorphic functions on the unit disc satisfying Borel's identity. This talk is based on the preprint: arXiv:2304.05715.

Toshiki Mabuchi (Osaka University)

Title: Kodaira dimension for Kaehlerian deformation

**abstract:** In this talk, as a first step to study the deformation invariance problem of plurigenera, we clarify the behavior of the Kodaira dimension under deformation of Kaehler manifolds.