The 30th Symposium of Complex Geometry (Kanazawa) 2024

	November 12	November 13	November 14	November 15
9:15-9:30	tea	tea	tea	tea
9:30-10:30	Ohsawa	Tsuji	Matsumura	Noguchi
10:50-11:50	R. Kobayashi	Inoue	Xiaojun Wu	Mabuchi
Lunch				
13:45-14:00	tea	tea	tea	
14:00-15:00	Y.Watanabe	Nitta	Atsuji	
15:20-16:20	Miyatake	Aoi	Katsuta	
16:40-17:40	Ono	Yonehara	R. Murakami (16:40-17:20)	

Venue: Shinoki geihnkan （しいのき迎賓館）金沢

November 12

9:30- 10:30 Takeo Ohsawa (Nagoya University)
Title:On Stein neighborhoods of certain Stein domains

10:50-11:50 R. Kobayashi (Nagoya University)
Title: Mean distribution of the ramification divisors associated to random projections

14:00-15:00 Yuta Watanabe (Chuo University)
Title: Singular Nakano positivity of direct image sheaves of adjoint bundles

15:20-16:20 Natsuo Miyatake (Tohoku University)
Title: Shannon entropy for harmonic metrics on cyclic Higgs bundles

16:40-17:40 Takashi Ono (The University of Osaka)
Title: Moduli Spaces of the Basic Hitchin equations on Sasakian three-folds

November 13

9:30- 10:30 Hajime Tsuji (Sophia University)
Title: Invariance of the pseudoeffectivity of canonical bundles under Kähler deformation

10:50-11:50 Eiji Inoue (Kyoto University)
Title: Higher moments and Distortion of pluri-subhamonic function on Kählerian space-time

14:00-15:00 Yasufumi Nitta (Tokyo University of Science)
Title: Extremal Kähler metrics and Mabuchi solitons on Fano manifolds

15:20-16:20 Takahiro Aoi (National Institute of Technology, Wakayama College)
Title: Poincaré type Kähler metrics and compactness of relative entropy

16:40-17:40 Shuhei Yonehara (The University of Osaka)
Title: Mikami-Weinstein type theorem for cosymplectic groupoid actions

November 14

9:30- 10:30 Shin-ichi MATSUMURA (Tohoku University)
Title: The nonvanishing problem for varieties with nef anticanonical bundle

10:50-11:50 Xiaojun WU (Universite Cote d'Azur Nice University)
Title: Compact Kaehler threefold with nef anticanonical line bundle

14:00-15:00 Atsushi Atsuji (Keio University)
Title: Analogues to Nevanlinna theory on discrete spaces.

15:20-16:20 Shuhei Katsuta (Nagaoya University)
Title: On Weyl's Peculiar Relation

16:40-17:20 Rei Murakami (Tohoku University)
Title: Weak limits of the J-flow and the cotangent flow

November 15

9:30- 10:30 Junjiro Noguchi (The University of Tokyo)
Title: Some remarks on basic materials in several complex variables

10:50-11:50 Toshiki Mabuchi (The University of Osaka)
Title: On the deformation invariance problem of plurigenera for Kähler manifolds

Abstracts

Nov. 12

Takeo Ohsawa (Nagoya University)

Title: On Stein neighborhoods of certain Stein domains

abstract:

Let X be a complex manifold which is holomorphically embedded in a complex manifold U as a closed submanifold. A well-known theorem of Siu [S] says that X admits a Stein neighborhood system in U if X is Stein.

In view of Siu's theorem, it may be worthwhile to study the existence of smoothly bounded Stein neighborhoods of X when U is a bounded domain in a complex manifold M and $X = U \cap Y$ for some complex submanifold Y of M. Such a question is closely related to the study of Dirichlet problem and $\bar{\partial}$-Neumann problem. Siu's method may be regarded as a natural extension of the observation that one can find a smoothly bounded strongly pseudoconvex neighborhood system of X in M if X is a strongly pseudoconvex domain in Y.

The purpose of this talk is to outline a proof of the following.

THEOREM 0.1. *There exists a two-dimensional compact complex manifold Y holomorphically embedded in $\mathbb{CP}^5$ and a Stein domain $X \subset Y$ with real-analytic boundary such that X admits a Stein neighborhood system with a "log-Hölder continuous" boundary in $\mathbb{CP}^5$ but does not admit a Stein neighborhood system with Lipschitz continuous boundary.*

In this example, X is the domain defined by

$$\{(\zeta, [z]); \mathrm{Re}(\zeta e^{2\pi i \mathrm{Re} z}) < 0\}$$

in the Veronese embedded product of $\mathbb{CP}^1 = \{\zeta \in \hat{\mathbb{C}} = \mathbb{C} \cup \{\infty\}\}$ and an elliptic curve $A = \{[z] = z + \mathbb{Z} + i\mathbb{Z}; z \in \mathbb{C}\}$. For the proof of the first part, Siu's proof is refined by the L^2 method. The latter part is an application of Harrington's result [H] asserting that Lipschitz pseudoconvex domains in $\mathbb{CP}^n$ admit bounded plurisubharmonic exhaustion functions, which is a refinement of Takeuchi's theorem [T] as well as [Oh-S]. A similar construction was given in [D-Oh] which does not assert the Hölder continuity.

References

[D-Oh] Diederich, K. and Ohsawa, T., *On pseudoconvex domains in $\mathbb{P}^n$,* Tokyo J. Math. **21** (1998), no. 2, 353-358.

[H] Harrington, P. S., *Bounded plurisubharmonic exhaustion functions for Lipschitz pseudoconvex domains in* $\mathbb{CP}^n$*,* J. Geom. Anal. **27** (2017), no. 4, 3404-3440.

[Oh-1] Ohsawa, T., *A Stein domain with smooth boundary which has a product structure,* Publ. Res. Inst. Math. Sci. **18** (1982), no. 3, 1185-1186.

[Oh-2] ——, *On Stein neighborhoods of certain Stein domains,* preprint.

[Oh-S] Ohsawa, T. and Sibony, N., *Bounded p.s.h. functions and pseudoconvexity in Kähler manifold,* Nagoya Math. J. **149** (1998), 1-8.

[S] Siu, Y.- T., *Every Stein subvariety admits a Stein neighborhood,* Invent. Math. **38**(1976/77), no.1, 89-100.

[T] Takeuchi, A., *Domaines pseudoconvexes infinis et la métrique riemannienne dans un espace projectif,* J. M. Soc. Japan 16(1964), 159-181.

Ryoichi Kobayashi (Nagoya University)

Title: Mean distribution of the ramification divisors associated to random projections

abstract:

Let X be a smooth projective variety over $\mathbb{C}$ and $(L, e^{-\varphi})$ an ample line bundle with a Hermitian metric s.t. $\omega = dd^c\varphi$ is a Kähler metric. For $m \gg 1$, mL is very ample. Let Z_μ denote the linear subspace of $|mL|^*$ of codimension $n+1$ whose defining equation is given by an element μ of $\mathbb{G}(n, |mD|)$. Then $\mathbb{G}(n, |mL|)$ is identified with the space of all linear projections associated to the Kodaira embedding of X into $|mL|^*$. If $X \cap Z_\mu = \emptyset$ (generic case) then $\mu : X \to \mathbb{P}^n$ is a finite morphism and therefore the Riemann-Hurwitz formula $R_\mu = \mu^* K_{\mathbb{P}^n}^{-1} \otimes K_X$ holds. This talk is concerned with the mean distribution of the ramification divisors $\{R_\mu\}_{\mu \in \mathbb{G}(N, |mD|)}$ on X when m becomes indefinitely large. I will show

$$\lim_{m\to\infty} \mathfrak{M}_{\mu \in \mathbb{G}(n,|mL|)} \frac{1}{(n+1)m} (\text{current of integration associated to } R_\mu) = \omega \ .$$

Interesting geometry arises in the case when the totality $\mathcal{G}_m$ of $\mu \in \mathbb{G}(n, |mL|)$ for which $|d\mu| \equiv 0$ is not empty.

Yuta Watanabe (Chuo University)

Title: Singular Nakano positivity of direct image sheaves of adjoint bundles

abstract: In this talk, we consider a proper Kähler fibration $f : X \to Y$ and a singular Hermitian line bundle (L, h) on X with semi-positive curvature. We prove that

the canonical L^2-metric on the direct image sheaf $f_*(\mathcal{O}_X(K_{X/Y}+L)\otimes\mathcal{I}(h))$ is singular Nakano semi-positive in the sense that the $\overline{\partial}$-equation can be solved with optimal L^2-estimate. Our proof does not rely on the theory of Griffiths positivity for the direct image sheaf.

Natsuo Miyatake (Tohoku University)
Title: Shannon entropy for harmonic metrics on cyclic Higgs bundles
abstract: Let X be a Riemann surface, and let $K_X \to X$ denote the canonical bundle. For each integer $r \geq 2$, each $q \in H^0(K_X^r)$, and each choice of a square root $K_X^{1/2}$ of the canonical bundle, we can canonically obtain a Higgs bundle, known as a cyclic Higgs bundle. A diagonal harmonic metric $h = (h_1, \ldots, h_r)$ on a cyclic Higgs bundle yields $r-1$ Hermitian metrics $H_1, \ldots, H_{r-1}$ on $K_X^{-1} \to X$, defined as $H_j = h_j^{-1} \otimes h_{j+1}$ for each $j = 1, \ldots, r-1$, while h_1, h_r, and q yield a degenerate Hermitian metric H_r on $K_X^{-1} \to X$. The Kähler metrics induced by these Hermitian metrics include flat Gaussian curvature Kähler metrics and constant negative Gaussian curvature Kähler metrics as special cases of q. The Hitchin equation for diagonal harmonic metrics on cyclic Higgs bundles can be viewed as a common generalization of the equations for constant negative and flat Gaussian curvature Kähler metrics. In this presentation, I will introduce a function, called entropy, to quantify the deviation of the Kähler metrics, constructed from the harmonic metrics on cyclic Higgs bundles, from the two aforementioned constant Gaussian curvature Kähler metrics. I will provide estimates for the entropy, extend it to more general weights, and compute the $r \to \infty$ limit in the case where q is identically zero.

Takashi Ono (The university of Osaka)
Title: Moduli Spaces of the Basic Hitchin equations on Sasakian three-folds
abstract: In this talk, I will introduce an equation called the basic Hitchin equation on Sasakian threefolds. This is a Sasakian analog of the Hitchin equation. I will show the moduli space admits a hyperkahler structure and explain why we can regard this space as the moduli space of the stable basic Higgs bundle of degree 0 and simple representation of the fundamental group.

Nov. 13

Hajime Tsuji (Sophia University)
Title: Invariance of the pseudoeffectivity of canonical bundles under Kähler deformation
abstract: In this talk, I will give a proof of the invariance of pseudoeffecivity of canonical bunldes under Kähler deformaiions. For the projective deformation, this can be proven by using L^2-extension theorem or Mori-Miyaoka's theirem which proves the uniruledness of a smooth projective variety with nonpseudoeffective canonical bunde In the case of Kähler deformation we prove the invariance by using Kähler-Ricci flows.

Eiji Inoue (Kyoto University)
Title: Higher moments and Distortion of pluri-subhamonic function on Kählerian space-time
abstract: A plurisubharmonic function on $X \times \triangle^*$ can be regarded as a ray of plurisubharmonic function on X and is related to test configuration. This object, which we call psh on Kählerian spacetime, is important in geometry as algebro-geometric information of certain "destabilizing" families of Kähler metrics is encoded into such object. Higher moments are important quantities of this object which plays a role of rulers for measuring "destabilizing property". A new notion called distortion allows us to create a new psh Dist(ϕ) on Kählerian spacetime which encodes higher moments of the original psh ϕ into the first moment of Dist(ϕ). One important application of this general study is the existence and uniqueness of optimal destabilizer in a certain setup related to Perelman entropy.

Yasufumi Nitta (Tokyo University of Science)
Title: Extremal Kähler metrics and Mabuchi solitons on Fano manifolds
abstract: In this talk, we concern relation between two kinds of canonical Kähler metrics on Fano manifolds, the Calabi's extremal Kähler metrics and the Mabuchi solitons. These are both generalizations of the concept of Kähler-Einstein metrics. It is known that the existence of Mabuchi solitons implies that of extremal Kähler metrics representing the first Chern class. It is also known that the converse is true for Fano manifolds of dimension up to two. Based on the above, we present examples of Fano manifolds in ALL dimensions greater than two which admit extremal Kähler metrics in every Kähler class, but do not admit Mabuchi solitons. A key concept is a holomorphic invariant of Fano manifolds called the Mabuchi constant, which is smaller than 1 for Fano manifolds admitting a Mabuchi soliton. Furthermore, we show that for Fano manifolds whose Mabuchi constants are smaller than 1, the existence of extremal Kähler metrics representing the first Chern class

implies that of Mabuchi solitons. This is partly joint work with Shunsuke Saito, and also partly joint work with Vestislav Apostolov and Abdellah Lahdili.

Takahiro Aoi (National Institute of Technology, Wakayama College)
Title: Poincaré type Kähler metrics and compactness of relative entropy
abstract: I will talk about a certain integrability result of plurisubharmonic functions for some singular measure with L^1-density. In the proof, the integrability theorem of Skoda-Zeriahi and the Ohsawa-Takegoshi L^2-extension theorem of holomorphic functions play a very important role. In addition, we show some compactness of relative entropy for Kähler metrics of Poincaré type on the finite energy space. This work is motivated by the variational characterization of constant scalar curvature Kähler (cone) metrics by Chen-Cheng and K.Zheng.

Shuhei Yonehara (The university of Osaka)
Title: Mikami-Weinstein type theorem for cosymplectic groupoid actions
abstract: The Mikami-Weinstein theorem is a generalization of the classical Marsden-Weinstein symplectic reduction theorem to the case of symplectic groupoid actions. In this talk, we introduce the notion of cosymplectic groupoid actions on cosymplectic manifolds and prove a theorem which is a natural analogue of the Mikami-Weinstein theorem.

Nov. 14

Shin-ichi MATSUMURA (Tohoku University)
Title: The nonvanishing problem for varieties with nef anticanonical bundle
abstract: In this talk, I discuss the nonvanishing problem in the framework of the "generalized" Minimal Model Program. I first explain a structure theorem for maximally rationally connected fibrations of projective klt pairs with nef anticanonical divisor, which generalizes Cao-Horing's result for smooth projective varieties. I also show that this structure theorem reduces the nonvanishing problem for nef anticanonical divisors to the rationally connected varieties and that the numerical class of the nef anticanonical bundle of projective 3-folds is represented by an effective divisor. The first part of this talk is joint work with Juanyong Wang (Chinese Academy of Sciences) and the latter part is joint work with Thomas Peternell (Bayreuth), Vladimir Lazic, Nikolaos Tsakanikas, Zhixin Xie (Saarbrucken).

Xiaojun WU (Universite Cote d'Azur Nice University)
Title: Compact Kaehler threefold with nef anticanonical line bundle
abstract: In this talk, I will talk about one recent joint work with Shin-ichi Matsumura on compact Kaehler threefold with nef anticanonical line bundle. The endeavor to classify compact Kaehler manifolds, particularly those exhibiting positive tangent bundles or anticanonical line bundles, constitutes a significant pursuit within mathematical discourse. While notable progress has been achieved, particularly through the seminal work of Cao and Horing in delineating the classification of projective manifolds with nef anticanonical line bundles, the analogous task in the compact Kaehler setting presents formidable challenges. In dimension three, based on Kaehler MMP developed by Horing-Peternell, a classification can be given using analytic tools like Segre currents, Q-conic bundle, orbifold vector bundle, etc.

Atsushi Atsuji (Keio University)
Title: Analogues to Nevanlinna theory on discrete spaces
abstract: In this talk we give some basic ideas to obtain an analogue to classical Nevanlinna theory on discrete spaces. One dimensional tropical Nevanlinna theory is known as an analogue to classical Nevanlinna theory for tropical meromorphic functions, and has been discussed by such authors as Halburd-Southall, Laine-Tohge and Korhonen-Laine-Tohge. One dimensional tropical Nevanlinna theory considers piecewise linear continuous functions on real numbers which are called tropical meromorphic functions. These functions can be naturally regarded as the functions on linear metric graphs. Then we wish to extend their arguments to the case of general metric graphs (i.e. cable systems). Our first topic in this talk is a generalization of one dimensional tropical Nevanlinna theory for tropical meromorphic functions on metric graphs. In the second topic we are talking about an analogue to Nevanlinna theory for harmonic morphisms between weighted infinite graphs. Our basic ideas of methods with stochastic processes on these discrete spaces will be introduced in order to obtain these analogues.

Shuhei Katsuta (Nagaoya University)
Title: On Weyl's Peculiar Relation
abstract: H. and J. Weyl discovered an intriguing relationship between holomorphic curves and their derived curves, which they referred to as "a peculiar relation" in their book '*Meromorphic Functions and Analytic Curves*'. In this talk, we will begin with a brief survey of classical Nevanlinna theory, and then discuss the generalization of this peculiar relation. As an example, we will observe how this result can be interpreted as a

combinatorial inequality.

Rei Murakami (Tohoku University)
Title: Weak limits of the J-flow and the cotangent flow
abstract: The J-equation and the Leung-Yau-Zaslow equation are fully nonlinear PDEs related to canonical metrics in Kaehler geometry. The solvability of the equations is known to be equivalent to certain numerical positivity. In this talk, we consider when the positivity does not hold. In particular, we study weak limits of the corresponding geometric flows on compact Kaehler surfaces.

Nov. 15

Junjiro Noguchi (The University of Tokyo)
Title: Some remarks on basic materials in several complex variables
abstract: I will give some remarks on basic materials in function theory of several complex variabes at an introductory text level. Writing three books, I have noticed some basic facts that I did not have had recognized. The first such example will be Oka'S 2nd Coherence Theorem; after defining the ideal sheaf for an arbitrary set, it should claim that the ideal sheaf of a closed set is coherent if and only if the set is analytic. I will discuss several others, and also a similarity in the two methods due to Oka and Grauert to solve Levi's problem. Oka's integral equation of Fredholm 2nd type will be discussed.

Toshiki Mabuchi (Osaka University)
Title: On the deformation invariance problem of plurigenera for Kähler manifolds
abstract: This is a continuation of my last year's talk on the deformation invariance problem of plurigenera for Kähler manifolds. Our method basically depends on an analogue of the Deligne-Fujiki-Ueno injectivity applied to Levine's obstruction for deformation.