A Nakai–Moishezon type criterion for supercritical deformed Hermitian–Yang–Mills equation

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§1. Motivation

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(X, χ, Ω): Calabi–Yau mfd of $\dim_{\mathbb{C}} X = n$
(χ: Ricci-flat Kähler, Ω: nowhere-vanishing holomorphic n-form)

A submanifold $\Sigma \subset X$ of $\dim_{\mathbb{R}} \Sigma = n$ is **Lagrangian** $:\Longleftrightarrow \chi|_\Sigma = 0$.

Lagrangian $\Sigma \subset X$ is **special** $:\Longleftrightarrow \mathrm{Im}(e^{-\sqrt{-1}\vartheta_0}\Omega)|_\Sigma = 0$ $(\vartheta_0 \in \mathbb{R})$

Theorem (Harvey–Lawson'82)

Any sLag's are homologically volume minimizing.

Conjecture (Thomas–Yau'02)

A given Lagrangian $\Sigma \subset X$ can be deformed to a sLag by Hamiltonian deformations iff the Hamiltonian isotopy class $[\Sigma]$ is "stable".

§1. Motivation

X: compact cpx mfd with $\dim_{\mathbb{C}} X = n$ (where X does **not** have to be CY)
$\alpha, \beta \in H^{1,1}(X, \mathbb{R})$ (β is Kähler), $\chi \in \beta$

Definition

$\omega \in \alpha$ is **deformed Hermitian–Yang–Mills (dHYM)**: $\Longleftrightarrow$

$$\mathrm{Im}\left(e^{-\sqrt{-1}\theta_0}(\omega + \sqrt{-1}\chi)^n\right) = 0 \Longleftrightarrow \sum_{i=1}^{n} \mathrm{arccot}(\lambda_i) = \theta_0 \pmod{2\pi}$$

where $\theta_0 \in \mathbb{R}$, $\lambda_1 \leqslant \ldots \leqslant \lambda_n$ are eigenvalues of $\omega_{i\bar{j}}\chi^{k\bar{j}}$.

Integrating the dHYM equation over X yields

$$\theta_0 = \arg\left(\int_X (\omega + \sqrt{-1}\chi)^n\right) \pmod{2\pi}.$$

§1. Motivation

- It is possible that $\int_X (\omega + \sqrt{-1}\chi)^n = 0$ when $n > 2$ (e.g. $X = \mathbb{CP}^3 \sharp \overline{\mathbb{CP}^3}$).
- If $\omega_1 \in \beta$ (resp. $\omega_2 \in \beta$) is dHYM with constant phase θ_1 (resp. θ_2) then $\theta_1 = \theta_2$ (**lifted angle**).
- Is it possible to define the lifted angle **algebraically**? (an open question raised by Collins–Xie–Yau'17)

Theorem (Leung–Yau–Zaslow'01)

When $X \to B$ is a "SYZ fibration", the sLag equation for sections of $\widehat{X} \to B$ is equivalent to the dHYM equation on a line bundle $L \to X$.

§1. Motivation

The goal of this talk is to give a numerical necessary and sufficient condition for the existence a solution to dHYM equation, **which confirms the mirror version of Thomas–Yau conjecture.**

Plan of Talk:

§2. Collins–Jacob–Yau conjecture
§3. Main results
§4. Regularized maximum
§5. Proof of the main theorem

§2. Collins–Jacob–Yau conjecture

Supercritical Phase Condition

We define the **Lagrangian phase operator** $Q_\chi \colon X \to (0, n\pi)$ by

$$Q_\chi(\omega) := \sum_{i=1}^{n} \operatorname{arccot}(\lambda_i), \quad \omega \in \alpha$$

so that the dHYM equation is $Q_\chi(\omega) = \theta_0$. Now we assume that $\theta_0 \in (0, \pi)$ by adding integer multiples of 2π.

Definition

$\omega \in \alpha$ is **supercritical** $:\Longleftrightarrow Q_\chi(\omega) < \pi$.

Remark

- ω is supercritical $\Longrightarrow \lambda_2 \geqslant 0$.
- $Q_\chi(\omega) < \Theta_0 < \pi \Longrightarrow \lambda_1 \geqslant -C(\Theta_0)$.

§2. Collins–Jacob–Yau conjecture

Subsolutions

We define the operator $P_\chi \colon X \to (0, (n-1)\pi)$ by

$$P_\chi(\omega) := \max_{k=1,\dots,n} \sum_{i \neq k} \operatorname{arccot}(\lambda_i), \quad \omega \in \alpha.$$

Definition

$\omega \in \alpha$ is a subsolution $:\Longleftrightarrow P_\chi(\omega) < \theta_0$.

Theorem (Collins–Jacob–Yau'15)

supercritical dHYM $^\exists \omega \in \alpha \Leftrightarrow$ supercritical subsolution $^\exists \underline{\omega} \in \alpha$.

- The subsolution condition is **analytic** and hard to check in practice.
- It is unknown that the existence of a solution **depends only on** α, β.

Nakai–Moishezon criterion

Theorem (Nakai'63, Moishezon'64, Demailly-Păun'04)

X: smooth proj. var. with $\dim_{\mathbb{C}} X = n$
$\alpha \in H^{1,1}(X;\mathbb{R})$ is Kähler
$\Longleftrightarrow \alpha^m \cdot Y > 0$, ${}^{\forall}Y \subset X$: subvar. with $m := \dim_{\mathbb{C}} Y = 1, \dots, n$.

If X is not projective...
$X = \mathbb{C}^n/\Lambda$: a flat torus ($\Lambda \subset \mathbb{C}^n$: generic lattice)

$$H^{1,1}(X;\mathbb{R}) = \{H | H \text{ is a Hermitian form on } \mathbb{C}^n \text{ with constant coefficients}\}.$$

Let α be the cohomology class corresponding to H. Then

$$\alpha^n \cdot X > 0 \Longleftrightarrow \det(H) > 0.$$

§2. Collins–Jacob–Yau conjecture

Proposition (Collins–Jacob–Yau'15)

For supercritical $\omega \in \alpha$, the subsolution condition $P_\chi(\omega) < \theta_0$ is equivalent to

$$\mathrm{Re}(\omega + \sqrt{-1}\chi)^m - \cot(\theta_0)\mathrm{Im}(\omega + \sqrt{-1}\chi)^m > 0 \quad (m = 1, \ldots, n-1).$$

Conjecture (Collins–Jacob–Yau'15)

supercritical dHYM $^\exists\omega \in \alpha \Longleftrightarrow$

$$\left(\mathrm{Re}(\alpha + \sqrt{-1}\beta)^m - \cot(\theta_0)\mathrm{Im}(\alpha + \sqrt{-1}\beta)^m\right) \cdot Y > 0$$

for $^\forall Y \subset X$: subvar. of $m := \dim_{\mathbb{C}} Y = 1, \ldots, n-1$.

§2. Collins–Jacob–Yau conjecture

Remark

When X is a smooth projective surface, the above CJY conjecture is a direct consequence from the Nakai–Moishezon criterion.

It is known that the CJY conjecture holds in some cases:

- For smooth Kähler surfaces (Collins–Jacob–Yau'15)
- "A version of" CJY conjecture for compact Kähler manifolds (G. Chen'20)
- For 3-dimensional smooth projective varieties (Datar–Pingali'20)
- For smooth projective varieties of arbitrary dimension (**Chu–Lee–T'21**)

A version of CJY conjecture

Q. How to choose a correct lift of θ_0?

Definition

A familly of closed real $(1,1)$-forms $\omega_t \in \alpha_t$ $(t \in [0,\infty))$ is a **test family** $:\Longleftrightarrow$

1. $\omega_0 = \omega \in \alpha$.
2. $s < t \Rightarrow \omega_s < \omega_t$.
3. $^{\exists}T \geqslant 0$; $\omega_t > \cot(\frac{\theta_0}{n})\chi$ $(t \in [T,\infty))$.

ω_t: a test family, $Y \subset X$: subvar. with $m := \dim_{\mathbb{C}} Y$

$$F^{\mathrm{Stab}}_{\theta_0}(Y,\{\omega_t\},t) := \int_Y \big(\mathrm{Re}(\omega_t + \sqrt{-1}\chi)^m - \cot(\theta_0)\mathrm{Im}(\omega_t + \sqrt{-1}\chi)^m\big).$$

§2. Collins–Jacob–Yau conjecture

Definition

(X, α, β) is **uniformly stable** along ω_t
$:\Longleftrightarrow {}^{\exists}\epsilon > 0$ s.t. ${}^{\forall}Y \subset X$ subvar. with $m = \dim_{\mathbb{C}} Y = 1, \ldots, n$, ${}^{\forall}t \in [0, \infty)$,

$$F^{\mathrm{Stab}}_{\theta_0}(Y, \{\omega_t\}, t) \geqslant (n - m)\epsilon \int_Y \chi^m.$$

Theorem (Chen'20)

supercritical dHYM ${}^{\exists}\omega \in \alpha \Longleftrightarrow (X, \alpha, \beta)$ is uniformly stable along ${}^{\forall}\{\omega_t\}$.

(Strategy for proving Chen's theorem)
Under the uniform stability assumption along ω_t, consider the following continuity path (C_t) for $\widetilde{\omega}_t \in \alpha_t$:

$$\mathrm{Re}(\widetilde{\omega}_t + \sqrt{-1}\chi)^n - \cot(\theta_0)\mathrm{Im}(\widetilde{\omega}_t + \sqrt{-1}\chi)^n - \widetilde{c}_t\chi^n = 0, \quad \widetilde{c}_t \in \mathbb{R}.$$

We use the twisted version of the Collins–Jacob–Yau's criterion as follows:

Theorem (Chen'20)

If there exists supercritical $\underline{\omega} \in \alpha$ such that $P_\chi(\underline{\omega}) < \theta_0$, then there exists supercritical $\omega \in \alpha$ satisfying

$$\mathrm{Re}(\omega + \sqrt{-1}\chi)^n - \cot(\theta_0)\mathrm{Im}(\omega + \sqrt{-1}\chi)^n - c\chi^n = 0,$$

where the constant $c \geqslant 0$ is uniquely determined by α, β.

§2. Collins–Jacob–Yau conjecture

We can easily observe that:

- $\widetilde{c}_t \geqslant 0$ by uniform stability assumption, and $\widetilde{c}_0 = 0$ since $\alpha_0 = \alpha$.
- (C_t) admits a solution for all $t \geqslant T$.

Set

$$\mathcal{T} := \{t \in [0, \infty) | (C_t) \text{ admits a solution}\}.$$

Then what we have to show is

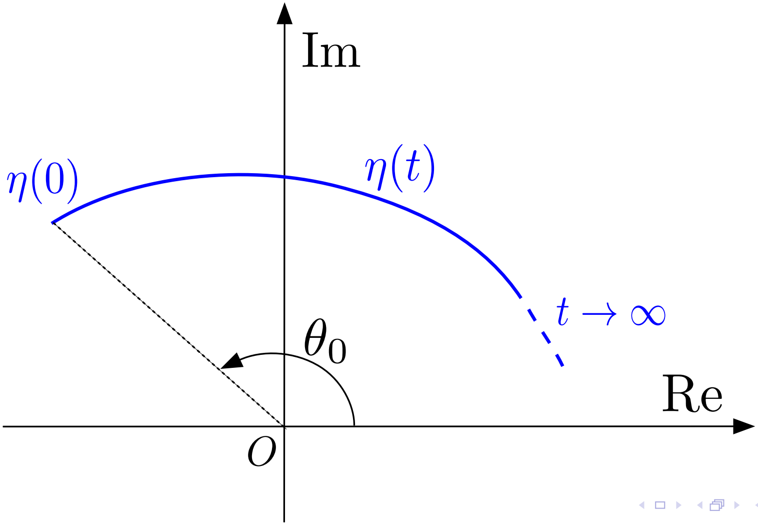
$$\mathcal{T} = [0, \infty).$$

Indeed, this is true, and the lifted angle is determined by

$$\theta_0 = (\text{Winding angle of } \eta(t) \ (t \in [0, \infty)),$$

where the path $\eta(t) := \int_X (\omega_t + \sqrt{-1}\chi)^n \subset \mathbb{C}$ does not path through the origin $0 \in \mathbb{C}$ by the uniform stability assumption.

§2. Collins–Jacob–Yau conjecture



Corollary (Chen'20)

The solvability of the dHYM equation does not depend on the choice of $\chi \in \beta$.

The remaining problems are summed up as follows:

Q. How to remove uniform constant ϵ?

Q. How to remove assumptions for test families ω_t?

§3. Main results

Removing the uniform constants

Definition

(X, α, β) is **stable** along a test family ω_t

$:\Longleftrightarrow$

$$F^{\mathrm{Stab}}_{\theta_0}(Y, \{\omega_t\}, t) \geqslant 0$$

for $^\forall Y \subset X$: subvar. with $m := \dim_{\mathbb{C}} Y = 1, \ldots, n$ and $^\forall t \in [0, \infty)$, with strict inequality holding if $m < n$.

Theorem (Chu–Lee–T'21)

supercritical dHYM $^\exists \omega \in \alpha \Longleftrightarrow (X, \alpha, \beta)$ is stable along $^\forall \{\omega_t\}$.

Removing test families

Corollary (Chu–Lee–T'21)

supercritical dHYM $^\exists\omega \in \alpha \iff$
$^\forall\gamma \in H^{1,1}(X;\mathbb{R})$: Kähler class, $^\forall Y \subset X$: subvar. with $m := \dim_{\mathbb{C}} Y = 1, \ldots, n$,
$^\forall k = 1, \ldots, m$

$$\left(\left(\mathrm{Re}(\alpha + \sqrt{-1}\beta)^k - \cot(\theta_0)\mathrm{Im}(\alpha + \sqrt{-1}\beta)^k\right) \cdot \gamma^{m-k}\right) \cdot Y \geqslant 0$$

with strict inequality holding if $m < n$.

§3. Main results

(Proof) For $\omega \in \alpha$ and a Kähler form $\sigma \in \gamma$, consider

$$\omega_t := \omega + t\sigma \quad (t \in [0, \infty)).$$

Then for $^\forall Y \subset X$ $(m = \dim_{\mathbb{C}} Y = 1, \dots, n)$ we have

$$\begin{aligned} &F^{\mathrm{Stab}}_{\theta_0}(Y, \{\omega_t\}, t) \\ &= \int_Y \left(\mathrm{Re}(\omega_t + \sqrt{-1}\chi)^m - \cot(\theta_0)\mathrm{Im}(\omega_t + \sqrt{-1}\chi)^m\right) \\ &= \sum_{k=0}^{m} t^{m-k} \binom{m}{k} \int_Y \left(\mathrm{Re}(\omega + \sqrt{-1}\chi)^k - \cot(\theta_0)\mathrm{Im}(\omega + \sqrt{-1}\chi)^k\right) \wedge \sigma^{m-k}. \end{aligned}$$

This is a polynomial of t with non-negative coefficients, and positive if $m < n$. $\square$

§3. Main results

The projective case

Corollary (Chu–Lee–T'21)

The CJY conjecture is true for all smooth projective varieties X.

(Proof) In the previous corollary we set $\gamma := c_1(L)$ (L: a very ample line bundle). Then for $^\forall Y \subset X$: subvar. with $m := \dim_{\mathbb{C}} Y$, $^\forall k = 1, \dots, m$, generic members $H_1, \dots, H_{m-k} \in |L|$, $Y \cap H_1 \cap \dots \cap H_{m-k}$ is a subvar. of dimension k in X.

$$\left(\left(\mathrm{Re}(\alpha + \sqrt{-1}\beta)^k - \cot(\theta_0)\mathrm{Im}(\alpha + \sqrt{-1}\beta)^k\right) \cdot \gamma^{m-k} \right) \cdot Y$$
$$= \left(\mathrm{Re}(\alpha + \sqrt{-1}\beta)^k - \cot(\theta_0)\mathrm{Im}(\alpha + \sqrt{-1}\beta)^k\right) \cdot (Y \cap H_1 \cap \dots \cap H_{m-k})$$

This is non-negative, and positive if $m < n$. □

§4. Regularized maximum

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For $\eta = (\eta_1, \ldots, \eta_N) \in (0, \infty)^N$ we define the function $M_\eta \colon \mathbb{R}^N \to \mathbb{R}$ by

$$M_\eta(t_1, \ldots, t_N) := \int_{\mathbb{R}^N} \max\{t_1 + h_1, \ldots, t_N + h_N\} \prod_{j=1,\ldots,N} \theta\left(\frac{h_j}{\eta_j}\right) dh_1 \ldots dh_N.$$

where θ denotes a non-negative smooth function on $\mathbb{R}$ with support in $[-1, 1]$.
Then M_η satisfies

1. $M_\eta(t_1, \ldots, t_N)$ is non-decreasing in all variables and convex on $\mathbb{R}^N$.
2. $M_\eta(t_1 + a, \ldots, t_N + a) = M_\eta(t_1, \ldots, t_N) + a$ for all $a \in \mathbb{R}$.

In particular, we have

$$\frac{\partial M_\eta}{\partial t_j} \geqslant 0, \quad \sum_{j=1} \frac{\partial M_\eta}{\partial t_j} = 1.$$

§4. Regularized maximum

$\{\Omega_j\}_{j=1,\ldots,N}$: a family of domains in X
φ_j: a smooth function on Ω_j satisfying:

1. $\varphi_j(x) < \max_{k=1,\ldots,N}\{\varphi_k(x)\}$ on each $x \in \partial\Omega_j$ (**gluing condition**)
2. $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi_j) < \theta_0$ on Ω_j

We choose a sufficiently small vector η so that $\varphi_j + \eta_j \leqslant \max_{k=1,\ldots,N}\{\varphi_k(x) - \eta_k\}$, and set $\varphi := M_\eta(\varphi_1, \ldots, \varphi_N)$. Then φ is smooth and

$$\frac{\partial^2 \varphi}{\partial z^k \partial z^{\bar{\ell}}} = \sum_{a,b} \frac{\partial^2 M_\eta}{\partial t_a \partial t_b} \cdot \frac{\partial \varphi_a}{\partial z^k} \cdot \frac{\partial \varphi_b}{\partial z^{\bar{\ell}}} + \sum_a \frac{\partial M_\eta}{\partial t_a} \cdot \frac{\partial^2 \varphi_a}{\partial z^k \partial z^{\bar{\ell}}}$$

$$\Longrightarrow \omega + \sqrt{-1}\partial\bar{\partial}\varphi \geqslant \sum_j \frac{\partial M_\eta}{\partial t_j} \cdot (\omega + \sqrt{-1}\partial\bar{\partial}\varphi_j)$$

$$\Longrightarrow Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi) \leqslant Q_\chi\left(\sum_j \frac{\partial M_\eta}{\partial t_j} \cdot (\omega + \sqrt{-1}\partial\bar{\partial}\varphi_j)\right) < \theta_0.$$

§5. Proof of the main theorem

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- The proof of Chen's theorem is based on induction of the dimension of a compact Kähler **manifold** X.
- The proof of our theorem is based on induction of the dimension of (possibly singular) **subvarieties** $Y \subset X$.

For a subvariety $Y \subsetneq X$ (resp. $Y = X$), let $\Gamma(Y)$ be the set of germs of smooth functions φ satisfying $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi) < \theta_0$ (resp. $P_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi) < \theta_0$ and $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi) < \pi$) near Y. Note that $\Gamma(Y)$ **is well-defined even if** Y **has singularities.**

By Collins–Jacob–Yau's criterion, it is enough to show that:

Theorem (Chu–Lee–T'21)

(X, α, β) is stable along $\{\omega_t\} \Longrightarrow \Gamma(Y) \neq \emptyset$ ($^\forall Y \subset X$: subvar.)

In what follows, we only consider the case $Y \subsetneq X$ for simplicity.

§5. Proof of the main theorem

Step 1. (Constructing subsolution on Y_{reg})

Theorem (Chu–Lee–T'21)

There exists a smooth function φ_Y on Y_{reg} such that

1. $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi_Y) < \theta_0$ on Y_{reg}.
2. $\varphi_Y \to -\infty$ along Y_{sing}.

(Proof) $\Phi\colon \widehat{X} \to X$: the resolution of singularities of an m-dimensional subvariety Y (a composition of blowups along smooth centers)
$\widehat{Y} := \Phi^{-1}(Y)$, E_0: exceptional divisor

Take a fiber metric h_{E_0} on $[E_0]$ and $\kappa_0 > 0$ such that $\xi := \Phi^*\chi - \kappa_0 F_{h_{E_0}} > 0$ on $\widehat{X}$.
For $0 < \varrho \ll 1$, we define

$$\widehat{\alpha}_{t,\varrho} := \Phi^*\alpha_t + \varrho t[\xi], \quad \widehat{\chi}_{t,\varrho} := \Phi^*\chi + (\varrho t)^n \xi.$$

§5. Proof of the main theorem

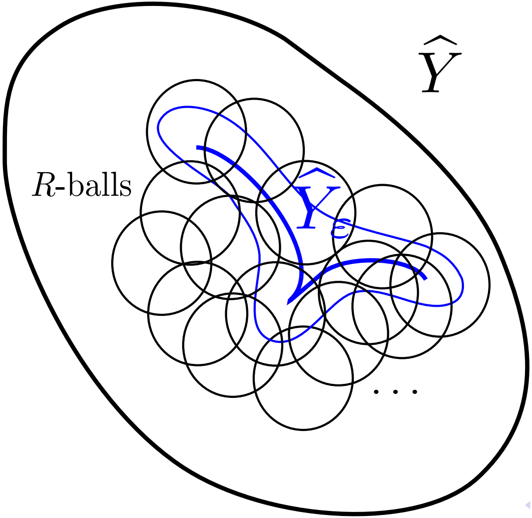
Consider the continuity path $(\widehat{C}_t)$ for $\widehat{\omega}_{t,\varrho} \in \widehat{\alpha}_{t,\varrho}$:

$$\mathrm{Re}(\widehat{\omega}_{t,\varrho} + \sqrt{-1}\widehat{\chi}_{t,\varrho})^m - \cot(\theta_0)\mathrm{Im}(\widehat{\omega}_{t,\varrho} + \sqrt{-1}\widehat{\chi}_{t,\varrho})^m - \widehat{c}_{t,\varrho}\widehat{\chi}_{t,\varrho}^m = 0, \quad \widehat{c}_{t,\varrho} \in \mathbb{R}.$$

Then we can show that

- $\widehat{c}_{t,\varrho} \geqslant 0$ and $(\widehat{C}_t)$ is solvable for all $t \in (0,1]$ and $0 < \varrho \ll 1$.
- The regularization (local smoothing & regularized maximum) of the weak limit $\widehat{\omega}_{0,0} := \lim_{t,\varrho\to 0} \widehat{\omega}_{t,\varrho}$ satisfies the desired conditions on $Y_{\mathrm{reg}} = \Phi(\widehat{Y} \backslash E_0)$ **if** $\widehat{\omega}_{0,0}$ **has zero Lelong numbers**.
- If not, we consider the ε-Lelong number sublevel set $\widehat{Y}_\varepsilon$ (for a suitable choice of $\varepsilon > 0$). Then $\Gamma(\Phi(\widehat{Y}_\varepsilon)) \neq \emptyset$ **by induction hypothesis.** Thus we may add the pullback of an element in $\Gamma(\Phi(\widehat{Y}_\varepsilon))$ when taking the regularized maximum. □

§5. Proof of the main theorem



Approximation of PSH functions

For a PSH function u on $B_{4R}(0) \subset \mathbb{C}^m$, $z \in B_{3R}(0)$ and $r \in (0, R/2)$ we define

$$u^{(r)}(z) := \int_{\mathbb{C}^n} r^{-2m} \rho\left(\frac{|y|}{r}\right) u(z-y)dy, \quad u_r(z) := \sup_{B_r(z)} u,$$

where $\rho(t)$ is a smooth non-negative function with support in $[0,1]$.

$$\nu_u(z,r) := \frac{u_{\frac{3}{4}R}(z) - u_r(z)}{\log\left(\frac{3}{4}R\right) - \log r} \xrightarrow{r \to 0} \nu_u(z) \quad (\textbf{Lelong number of } u \textbf{ at } z)$$

In order to check the gluing condition, we use the following:

Lemma (Błocki–Kołoziej'07, Chen'19)

For any $r \in (0, R/2)$ and $z \in B_{3R}(0)$, the following estimates hold:

1. $0 \leqslant u_r(z) - u_{\frac{r}{2}}(z) \leqslant (\log 2)\nu_u(z, r)$
2. $0 \leqslant u_r(z) - u^{(r)}(z) \leqslant \eta\nu_u(z, r)$

where a constant $\eta > 0$ is defined by

$$\eta := \frac{3^{2m-1}}{2^{2m-3}} \log 2 + \mathrm{Vol}(\partial B_1(0)) \int_0^1 \log\left(\frac{1}{t}\right) t^{2m-1} \rho(t) dt.$$

Step 2. (Gluing argument) By induction hypothesis,
U_1: neighborhood of Y_{sing}, $\psi \in C^\infty(U_1, \mathbb{R})$ s.t. $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\psi) < \theta_0$.
Take open neighborhoods $U_3 \Subset U_2 \Subset U_1$ of Y_{sing} in X such that

1. $\varphi_Y > \psi + 2$ in $Y \cap (U_1 \backslash U_2)$
2. $\varphi_Y < \psi - 2$ in $Y \cap U_3$

We consider an extension

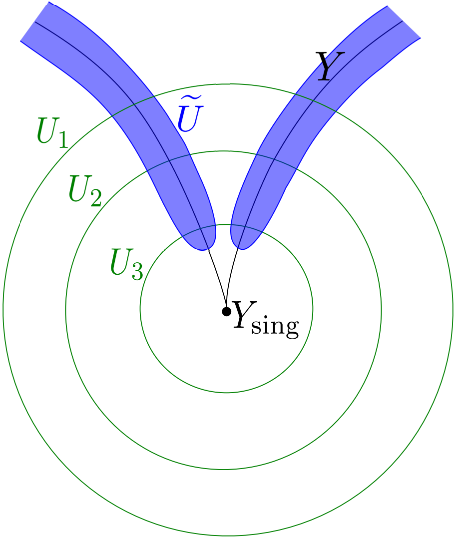
$$\widetilde{\varphi}_Y := \varphi_Y + A d_Y^2, \quad A \gg 0$$

Then we observe that $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\widetilde{\varphi}_Y) < \theta_0$ in an open neighborhood $\widetilde{U}$ of $Y \backslash W$ in X, and

1. $\widetilde{\varphi}_Y > \psi + 1$ in $\widetilde{U} \cap (U_1 \backslash U_2)$
2. $\widetilde{\varphi}_Y < \psi - 1$ in $\widetilde{U} \cap U_3$

Let φ be the regularized maximum of $(\widetilde{U}, \widetilde{\varphi}_Y)$ and (U_2, ψ). Then we conclude that $Q_\chi(\omega + \sqrt{-1}\partial\bar{\partial}\varphi) < \theta_0$ on a small neighborhood of Y in X.

§5. Proof of the main theorem



Thank you!