The 24th Symposium on Complex Geometry
Kanazawa 2018

	Nov.13	Nov.14	Nov.15	Nov.16
9:10-9:20	coffee	coffee	coffee	coffee
9:20-10:20	Mabuchi	Mok	Oguiso	Fujino
10:30-11:30	Ohsawa	R. Kobayashi	Tsuji	Goto
Lunch				
13:00-13:20	coffee	coffee	coffee	
13:20-14:20	Inoue	Odaka	Honda	
14:30-15:30	Tadano	Spotti	Kusakabe	
15:40-16:40	Kikuta	Hisamoto	Kasuya	
		18:00- reception		

Venue: Shiinoki Cultural Complex, Ishikawa Prefecture, 3F, Seminar Room B (Seminar Room A is only for the morning of November 15)

Nov. 13

9:20- 10:20 Toshiki Mabuchi (Osaka University)
Title: Extremal Kähler versions of the Yau-Tian-Donaldson Conjecture

10:30-11:30 Takeo Ohsawa (Nagoya University)
Title: Rigidity theorems by the L2 method

13:20-14:20 Eiji Inoue (The University of Tokyo)
Title: Kähler-Ricci soliton and moduli space of Fano manifolds

14:30-15:30 Homare Tadano (Tokyo University of science)
Title: Some Compactness Theorems for Complete Ricci Solitons

15:40-16:40 Shin Kikuta (Kougakuin University)
Title: Degeneration of positivity for log-canonical bundle and boundary behavior of Kähler-Einstein metric

Nov. 14

9:20- 10:20 Ngaiming Mok (The University of Hong Kong)
Title: Universal Covering Maps onto Finite-Volume Quotients of Bounded Symmetric Domains from the Perspective of Complex Differential Geometry

10:30-11:30 Ryoichi Kobayashi (Nagoya University)
Title: Rigidity of compact holomorphic curves in compact complex parallelizable manifolds $\Gamma\backslash SL(2,\mathbb{C})$

13:20-14:20 Yuji Odaka (Kyoto University)
Title: Collapsing Ricci-flat Kahler metrics and Moduli compactification

14:30-15:30 Cristiano Spotti (Aarhus University)
Title: Singular Kähler-Einstein metrics on the complex projective space

15:40-16:40 Tomoyuki Hisamoto (Nagoya University)
Title:The reduced J-functional and application to the Yau-Tian-Donaldson type conjecture

Nov. 15

9:20- 10:20 Keiji Oguiso (The University of Tokyo)
Title: A surface with discrete and non-finitely generated automorphism group and infinitely many real forms

10:30-11:30 Hajime Tsuji (Sophia University)
Title: The limit of Kähler-Ricci flows

13:20-14:20 Nobuhiro Honda (Tokyo Institute of Technology)
Title: Twistors, quartics, and del Pezzo fibrations

14:30-15:30 Yuta Kusakabe (Osaka University)
Title: Elliptic characterization and localization of Oka manifolds

15:40-16:40 Hisashi Kasuya (Osaka University)
Title: DGA-Models and constructions of variations of mixed Hodge structures

Nov. 16

9:20- 10:20 Osamu Fujino (Osaka University)
Title: Simple connectedness of Fano log pairs with semi-log canonical singularities

10:30-11:30 Ryushi Goto (Osaka University)
Title: Einstein-Hermitian vector bundles over generalized Kähler manifolds

ABSTRACT

Speaker: Toshiki Mabuchi (Osaka University)

Title: Extremal Kähler versions of the Yau-Tian-Donaldson Conjecture

abstract: For Kähler-Einstein metrics, the Yau-Tian-Donaldson Conjecture was recently solved affirmatively by Chen-Donaldson-Sun and Tian. However, the conjecture is still open for general polarizations or more generally for extremal Kähler cases. In this talk, we discuss extremal Kähler versions of the conjecture with emphasis on how to apply the peak section method to the problem.

Speaker: Takeo Ohsawa (Nagoya University)

Title: Rigidity theorems by the L^2 method

abstract: Geometric invariants of complex manifolds are encoded in the space of L^2 space of holomorphic sections of vector bundles. They are compressed in the Bergman kernel as the works of Kodaira, Hörmander and Fefferman have shown, so that relations between analysis and geometry on complex manifolds are suggested in the results on the Bergman kernels. Given an analytic family of complex manifolds, say $\pi : M \to T$, the parameter dependence of the Bergman kernel $K_t = K_{M_t}$ of $M_t = \pi^{-1}(t)$ reflects how the complex structure of M_t deforms. It is known by Berndtsson that $\log K_t$ depends plurisubharmonically in t if M is weakly 1-complete ($= C^\infty$ plurisubharmonic) and Kählerian. An immediate consequence of Berndtsson's theorem is that such a family is locally analytically trivial if $\log K_t \in C^\infty$ and $\partial\bar{\partial} \log K_t$ annihilates a horizontal distribution (a subbundle of $T_M^{1,0}$ which bijects to $T_T^{1,0}$ by π). This generalizes a result of Maitani and Yamaguchi for Stein families of Jordan domains. Roughly speaking, the Bergman kernel detects the rigidity of analytic families. On the other hand, it was proved by Nishino that a Stein submersion over the unit disc is trivial if the fibers are $\mathbb{C}$. Although this rigidity does not follow directly from $K_\mathbb{C} \equiv 0$, it turned out that an L^2 extension theorem in [Oh-T'87] is available to give its alternate proof (cf. [Oh'18-1]). For the family of $\mathbb{C}^n$ with $n \geq 2$, a rigidity criterion can be proved by a similar method (cf. [Oh'18-2]).

References

[Oh'18-1] Ohsawa, T., *L^2 proof of Nishino's rigidity theorem*, to appear in Kyoto J. Math.

[Oh'18-2] Ohsawa, T., *A generalization of Nishino's rigidity theorem*, preprint.

[Oh-T'87] Ohsawa, T. and Takegoshi, K., *On the extension of L^2 holomorphic functions*, Math. Z. 195 (1987), 197-204.

Speaker: Eiji Inoue (The University of Tokyo)

Title: Kähler-Ricci soliton and moduli space of Fano manifolds

abstract: In this talk, I will introduce the existence (and uniqueness) question on a moduli space of Fano manifolds admitting Kähler-Ricci solitons and discuss how we should formulate and can solve this question. The usual notion of deformation is inappropriate in our moduli problem and it will be replaced by "deformation with K-optimal action". I will also explain a new (almost) analytic construction of the moduli space without any stacky words, which is newly discovered by R. Dervan and P. Naumann after https://arxiv.org/abs/1802.08128.

Speaker: Homare Tadano (Tokyo University of science)

Title: Some Compactness Theorems for Complete Ricci Solitons

abstract: The aim of this talk is to discuss the compactness of complete Ricci solitons. Ricci solitons were introduced by R. Hamilton in 1982 and are natural generalizations of Einstein manifolds. They correspond to self-similar solutions to the Ricci flow and often arise as singularity models of the flow. The importance of Ricci solitons was demonstrated by G. Perelman, where they played crucial roles in his affirmative resolution of the Poincaré conjecture.

In this talk, after we reviewed basic facts on Ricci solitons, I would like to give some new compactness theorems for complete Ricci solitons. Our results are natural generalizations of the compactness theorems due to W. Ambrose (1957), J. Cheeger, M. Gromov, and M. Taylor (1982), M. Fernández-López and E. García-Río (2008), M. Limoncu (2010, 2012), Z. Qian (1997), Y. Soylu (2017), G. Wei and W. Wylie (2009), and S. Zhang (2014). Time permitting, I would like to extend such compactness theorems to the case of transverse Ricci solitons on complete Sasaki manifolds.

Speaker: Shin Kikuta (Kougakuin University)

Title: Degeneration of positivity for log-canonical bundle and boundary behavior of Kähler-Einstein metric

abstract: We would like to discuss some relations between the positivity of the logcanonical bundle and the boundary behavior of the Kähler- Einstein metric over a quasi-projective manifold. In this talk, we will propose a conjecture about them, and present supporting examples and our work in progress.

Speaker: Ngaiming Mok (The University of Hong Kong)

Title: Universal Covering Maps onto Finite-Volume Quotients of Bounded Symmetric Domains from the Perspective of Complex Differential Geometry

abstract: By the Uniformization Theorem a compact Riemann surface other than the Riemann Sphere or an elliptic curve is uniformized by the unit disk and equivalently by the upper half plane. The upper half plane is also the universal covering space of the moduli space of elliptic curves equipped with a suitable level structure. In Several Complex Variables, the Siegel upper half plane (which is biholomorphic to a bounded symmetric domain) is an analogue of the upper half plane, and it is the universal covering space of moduli spaces of polarized Abelian varieties with level structures. In general, finite-volume quotients of bounded symmetric domains, which are naturally quasi-projective varieties, are of immense interest to Several Complex Variables, Algebraic Geometry and Number Theory, and an important object of study is the universal covering map $\pi_\Gamma : \Omega \to X_\Gamma$ from a bounded symmetric domain Ω onto its quotient $X_\Gamma := \Omega/\Gamma$ by a torsion-free discrete lattice $\Gamma \subset \mathrm{Aut}(\Omega)$. We will explain a differential-geometric approach to the study of the universal covering map revolving around the notion of asymptotic curvature behavior, rescaling arguments and the use of meromorphic foliations, and illustrate how this approach using transcendental techniques leads to various results concerning totally geodesic subvarieties of finite-volume quotients without the assumption of arithmeticity.

Speaker: Ryoichi Kobayashi (Nagoya University)

Title: Rigidity of compact holomorphic curves in compact complex parallelizable manifolds $\Gamma\backslash\mathrm{SL}(2,\mathbb{C})$

abstract: Let $\Gamma \subset \mathrm{SL}(2,\mathbb{C})$ be a cocompact lattice and $X = \Gamma\backslash\mathrm{SL}(2,\mathbb{C})$ the associated compact complex parallelizable manifold. We show that any non-constant holomorphic map $f : M \to X$ from a compact Riemann surface M into a compact complex parallelizable manifold X is expressed as a composition $f = t \circ h \circ \alpha$ where $\alpha : M \to \mathrm{Alb}(M)$ is the Albanese map, $h : \mathrm{Alb}(M) \to X = \Gamma\backslash\mathrm{SL}(2,\mathbb{C})$ has its image in a maximal torus $T = \Gamma \cap A\backslash A \cong \mathbb{Z}\backslash\mathbb{C}^*$ in X defining an algebraic group homomorphism

$h: \mathrm{Alb}(M) \to T = (A \cap \Gamma) \backslash A$, and finally t is a right translation by some element of $\mathrm{SL}(2, \mathbb{C})$. The proof is based on Bishop's criterion of analyticity of sets combined with a simple observation in hyperbolic geometry. If time permits, I will discuss applications of this rigidity.

Speaker: Yuji Odaka (Kyoto University)
Title: Collapsing Ricci-flat Kahler metrics and Moduli compactification
abstract: We explain some parts of our recent program joint with Yoshiki Oshima (Osaka Univ), which provide a moduli-theoretic framework for the collapsing of Ricci-flat Kahler metrics by certain explicit compactifications of classical moduli varieties. The speaker originally called the obtained compactification "tropical geometric compactification" (available as arxiv:1406.7772, arXiv:1705.05545) and our joint work (cf., arXiv:1805.01724, 1810.07685) largely develops the theory.

Speaker: Cristiano Spotti (Aarhus University)
Title: Singular Kähler-Einstein metrics on the complex projective space.
abstract: I will discuss some properties of KE metrics with cone singularities along a non-necessarily smooth hypersurface of the complex projective space. The talk is based on some joint works with Martin de Borbon, Patricio Gallardo and Jesus Martinez-Garcia.

Speaker: Tomoyuki Hisamoto (Nagoya University)
Title: The reduced J-functional and application to the Yau-Tian-Donaldson type conjecture
abstract: We discuss the formulation of the Yau-Tian-Donaldson type conjecture when the automorphism group is non-discrete. The main points are around the G-reduced J-functional and the results are applied to two extreme cases: cscK metric for toric polarized manifolds and Mabuchi soliton for Fano manifolds.

Speaker: Keiji Oguiso (The University of Tokyo)
Title: A surface with discrete and non-finitely generated automorphism group and infinitely many real forms
abstract: This is a joint work with Professor Dinh at Singapore. We show that there is a smooth complex projective variety, of any dimension greater than or equal to two, whose automorphism group is discrete and not finitely generated. Moreover, this variety admits infinitely many real forms which are mutually non-isomorphic over $\mathbb{R}$. Our result is inspired by the work of Lesieutre and answers questions by Dolgachev, Esnault and Lesieutre.

Speaker: Hajime Tsuji (Sophia University)
Title: The limit of Kähler-Ricci flows
abstract: We consider the Kähler-Ricci flows on compact Kähler manifolds with pseudoeffective canonical bundles. If the canonical bundle is semiample, the limit of Kähler Ricci flow is known to be the Generalized Kähler-Einstein forms. by Song and Tian. We generalize this result to the case of abundant canonical bunldes. The method is the discretization of Kähler-Ricci flows.

Speaker: Nobuhiro Honda (Tokyo Institute of Technology)
Title: Twistors, quartics, and del Pezzo fibrations
abstract: I will talk about a recent result on algebraic description of a wide class of compact twistor spaces associated to self-dual metrics on 4-manifolds. Each of these spaces is birational to the total space of a Del Pezzo fibration over CP^1, and may be described by a single quartic polynomial of a particular form. Generic fibers of the fibration are (possibly singular) Del Pezzo surfaces of degree two.

Speaker: Yuta Kusakabe (Osaka University)

Title: Elliptic characterization and localization of Oka manifolds

abstract: Gromov's Oka principle gives a sufficient condition, called ellipticity, for a complex manifold to be Oka. It is a well-known problem whether the Oka property can be characterized by ellipticity or its variants. There is another ellipticity condition introduced by Gromov, which is called Condition Ell_1. We prove that Condition Ell_1 characterizes Oka manifolds. This characterization gives affirmative answers to Gromov's conjectures. As another application, we establish the localization principle for Oka manifolds, which gives new examples of Oka manifolds.

Speaker: Hisashi Kasuya (Osaka University)

Title: DGA-Models and constructions of variations of mixed Hodge structures

abstract: We define objects over Morgan's mixed Hodge diagrams which will be algebraic models of unipotent variations of mixed hodge structures over Kähler manifolds. As an analogue of Hain-Zucker's equivalence between unipotent variations of mixed Hodge structures and mixed Hodge representations of the fundamental group with Hain's mixed hodge structure, we give an equivalence between the category of our VMHS-like objects and the category of mixed Hodge representations of the dual Lie algebra of Sullivan's minimal model with Morgan's mixed Hodge structure. By this equivalence, we give techniques of constructing variations of mixed Hodge structures by using a volume form on compact Kähler manifolds. As an application, we show a precise relation between Morgan's mixed Hodge structure and Hain's mixed Hodge structure.

Speaker: Osamu Fujino (Osaka University)

Title: Simple connectedness of Fano log pairs with semi-log canonical singularities

abstract: It is well known that every Fano manifold is simply connected. In this talk, I will explain that any union of slc strata of a Fano log pair with semi-log canonical singularities is simply connected. In particular, Fano log pairs with semi-log canonical singularities are simply connected. This is a joint work with Wenfei Liu.

Speaker: Ryushi Goto (Osaka University)

Title: Einstein-Hermitian vector bundles over generalized Kähler manifolds

abstract: We discuss the correspondence between stable generalized holomorphic vector bundles and Einstein-Hermitian vector bundles over generalized Kähler manifolds of symplectic type. Kähler-Ricci solitons and Einstein Hermitian co-Higgs bundles give rise to examples of Einstein-Hermitian generalized vector bundles. The stability of Poisson modules on complex surfaces are also discussed.